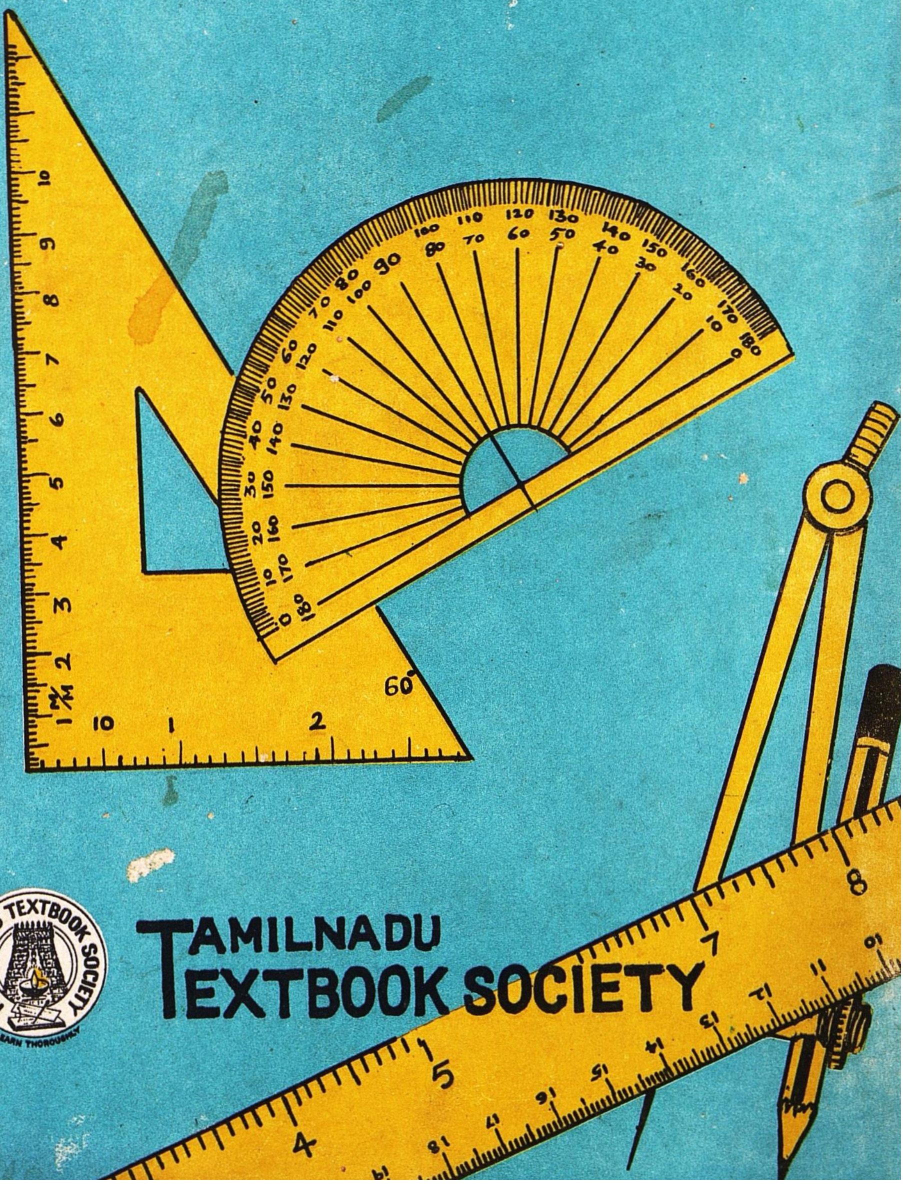
MATHEMATICS 7



TAMILNADU TEXTBOOK SOCIETY

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MATHEMATICS

VII STANDARD

Untouchability is a Sin
Untouchability is a Crime
Untouchability is inhuman

TAMILNADU
TEXTBOOK SOCIETY
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1. Number System

1-1. Natural Numbers and Whole Numbers

PRE-TEST

EXERCISE 1-1

1. What is the first member of natural numbers ?
2. Name the first whole number ?
3. Is there a Predecessor for 1 in the whole numbers ? If so write it.
4. 44*444 – State the place value of the stared digit.
5. Give the number name of 2 74 35 618 in Indian System.
6. Write the number name of 19 11 410 in the English system.
7. Write the Numeral of :

   One crore nine lakh eighty two thousand one hundred nine.
8. Write the Numeral of :

   Seven million six hundred eight thousand nine hundred eight.
9. Rewrite in expanded form :

   81099
10. Rewrite in expanded notation using exponents :

    46034
11. Give the number in standard form :

    $9 \times 10^6 + 7 \times 10^5 + 6 \times 10^3 + 2 \times 10^1 + 9.$
12. Arrange in descending order :

    3952, 3592, 3259, 3295, 3529
13. What number should be added to make 72895 a lakh ?

14. Simplify :

    $62 - (62-53)$

15. Find by short method : 927×1002.

16. When a number is divided by 362, the quotient is 301 and the remainder is 61. Find the number ?

17. Express 5055 correct to a hundred.

18. Simplify :

    $1000 - 8 \times 12 - 11 \times 15 - 552 \div 6$

1–2. Whole Numbers - Addition-(A)-Some Short Methods

Consider the following examples carefully.

Examples :

1. $68 + 42 = 100 + 10 = 110$
2. $57 + 63 = 110 + 10 = 120$
3. $194 + 26 = 210 + 10 = 220$
4. $271 + 329 = 590 + 10 = 600$

In the above examples, you can note that the digits in the unit place are the complements of 10. (Their sum is equal to 10.) Using this fact additions of such numbers can be done easily.

EXERCISE 1-2 (ORAL)

Find the Sum :

1. $31 + 49$
2. $62 + 78$
3. $23 + 37$
4. $74 + 66$
5. $35 + 65$
6. $199 + 71$
7. $128 + 132$
8. $237 + 343$
9. $356 + 274$
10. $625 + 445$

1-3. Whole Numbers – Addition (B)

Study the following examples :

Examples :

1. $75 + 47 = 75 + 45 + 2 = 120 + 2 = 122$
2. $46 + 35 = 46 + 34 + 1 = 80 + 1 = 81$
3. $97 + 44 = 97 + 43 + 1 = 140 + 1 = 141$
4. $68 + 65 = 68 + 62 + 3 = 130 + 3 = 133$

EXERCISE 1-3 (ORAL)

Using the Method above find the Sum :

1. $72 + 29$
2. $63 + 48$
3. $84 + 69$
4. $55 + 48$
5. $86 + 37$
6. $97 + 87$
7. $68 + 38$
8. $69 + 32$
9. $375 + 47$
10. $597 + 44$

1-4. Whole Numbers – Addition (C)

Look at the following examples :

Examples :

1. $64 + 43 + 46$
$= 64 + 46 + 43$
$= 110 + 43$
$=$ **153.**

2. $72 + 37 + 88$
$= 72 + 88 + 37$
$= 160 + 37$
$=$ **197.**

3. $93 + 25 + 47$
$= 93 + 47 + 25$
$= 140 + 25$
$=$ **165.**

4. $36 + 62 + 74 + 58$
$= 36 + 74 + 62 + 58$
$= 110 + 120$
$= \mathbf{230}$.

EXERCISE 1-4

Find the Sum :

1. $32 + 27 + 28$
2. $63 + 52 + 47$
3. $74 + 53 + 66$
4. $85 + 36 + 45$
5. $96 + 41 + 34$
6. $47 + 76 + 23$
7. $28 + 31 + 42 + 69$
8. $49 + 38 + 81 + 92$
9. $83 + 92 + 77 + 28$
10. $129 + 87 + 71 + 33$

2-1. Integers

Multiplication by 5 :

Let us recall what we have studied in earlier classes.

1. Find : $724 \times 5 = ?$ $\quad 5 = 10 \div 2$

$$\therefore \; 724 \times 5 = \frac{724 \times 10}{2} = \frac{7240}{2} = 3620$$

$(724 \div 2 = 362)$

2. Find the value of : 867×5

$$867 \times 5 = \frac{867 \times 10}{2} = \frac{8670}{2} = 4335$$

($867 \div 2$. The quotient is 433)

In general we have the following :

1. To multiply an even number by 5, we divide the number by 2 and a zero is placed at the unit's place of the quotient.

2. For multiplying an odd number by 5, the number is divided by 2 and 5 is placed at the unit's place of the quotient.

Now let us consider the sums worked above :

1. $724 \div 2 = 362$

   $\therefore \quad 724 \times 5 = 3620.$

2. $867 \div 2$. Here the quotient is 433, when 867 is divided by 2.

   $\therefore \quad 867 \times 5 = 4335.$

EXERCISE 2-1 (ORAL)

Multiply :

1. 24×5	6. 77×5
2. 38×5	7. 89×5
3. 72×5	8. 81×5
4. 96×5	9. 95×5
5. 63×5	10. 460×5

2-2. Whole Numbers - Even Numbers

Multiplication (by some of the multiples of 5) :

Examples :

1. $28 \times 15 = 14 \times 30 = 420$
2. $96 \times 15 = 48 \times 30 = 1440$
3. $148 \times 15 = 74 \times 30 = 2220$
4. $26 \times 35 = 13 \times 70 = 910$
5. $42 \times 45 = 21 \times 90 = 1890$

Here note that 28, 96, 148, 26, 42 are even numbers.

We multiply the number by 10 and add with it half of it.

Aliter : The first three multiplications can be done in another way.

1. $28 \times 15 = (28 + 14) \times 10 = 420$
2. $96 \times 15 = (96 + 48) \times 10 = 1440$
3. $148 \times 15 = (148 + 74) \times 10 = 2220$

We add half of the even number to the number and multiply it by 10.

EXERCISE 2-2 (ORAL)

Find the product of the following :

1.	42×15	8.	464×15
2.	84×15	9.	846×15
3.	90×15	10.	998×15
4.	76×15	11.	78×45
5.	68×15	12.	84×35
6.	120×15	13.	498×35
7.	222×15	14.	924×45

2–3. Whole Numbers — Odd Numbers

Let us learn how we could multiply an odd number by 15 easily.

Examples :

1. Find the value of : 67×15

   $67 \times 15 = 67 \times 10 + 67 \times 5 = 670 + 335 = 1005.$

   Step (1) $67 + 33 = 100$

   Step (2) 1005.

2. Find the value of : 83×15

   Step (1) $83 + 41 = 124$

   Step (2) 1245.

3. Find the value of : 95×15

   Step (1) $95 + 47 = 142$

   Step (2) 1425.

4. Find the value of : 41×15

   Step (1) $41 + 20 = 61$

   Step (2) 615.

EXERCISE 2–3

Examine the above examples and do the following sums.

1. 11×15	6. 91×15
2. 23×15	7. 103×15
3. 35×15	8. 215×15
4. 47×15	9. 327×15
5. 59×15	10. 439×15

2-4. Integers - Multiplication in Series

Note the following Multiplication Sums :

1. $15 \times 33 \times 48$
$= 15 \times 48 \times 33$
$= 15 \times 48 \times 3 \times 11$
$= 720 \times 3 \times 11$
$= 2160 \times 11$
$= 23760.$

2. $32 \times 15 \times 22$
$= 16 \times 30 \times 11 \times 2$
$= 480 \times 2 \times 11$
$= 960 \times 11$
$= 10560.$

EXERCISE 2–4

Find the Product of :

1. $62 \times 15 \times 44$	6. $86 \times 35 \times 5$
2. $34 \times 15 \times 55$	7. $98 \times 45 \times 12$
3. $63 \times 15 \times 66$	8. $84 \times 55 \times 16$
4. $76 \times 15 \times 5$	9. $92 \times 65 \times 24$
5. $98 \times 15 \times 33$	10. $66 \times 75 \times 28$

2 - 5.. Multiplication of Integers by 25

Let us recall what we have learnt in the earlier standards.

Examples :

1. Find the Value of : 924 × 25

   Step (1) 924 ÷ 4

   Quotient 231, Remainder 0

   Step (2) 924 × 25 = 23100.

2. Find the Value of : 1953 × 25

   Step (1) 1953 ÷ 4

   Quotient 488, Remainder 1

   Step (2) 1953 × 25 = 48825.

3. Find the Value of : 714 × 25

   Step (1) 714 ÷ 4

   Quotient 178, Remainder 2

   Step (2) 714 × 25 = 17850.

4. Find the Value of : 8947 × 25

   Step (1) 8947 ÷ 4

   Quotient 2236, Remainder 3

   Step (2) 8947 × 25 = 223675.

NOTE :

In the above illustrations you have seen that the given number is divided by 4.

(a) If the remainder is 0, write two zeroes to the right of the Quotient.

(b) If the remainder is 1, write 25 to the right of the Quotient.

(c) If the remainder is 2, write 50 to the right of the Quotient.

(d) If you get the remainder as 3, write 75 to the right of the Quotient.

EXERCISE 2–5 (Oral)

Find the Product of :

1. 841 × 25	6. 964 × 25
2. 953 × 25	7. 985 × 25
3. 648 × 25	8. 1237 × 25
4. 532 × 25	9. 2539 × 25
5. 626 × 25	10. 840 × 25

2-6. Quick Multiplication by 11

Study the following Patterns :

$11 \times 11 = 1\,(1 + 1)\,1 = 121$

$12 \times 11 = 1\,(2 + 1)\,2 = 132$

$13 \times 11 = 1\,(3 + 1)\,3 = 143$

$14 \times 11 = 1\,(4 + 1)\,4 = 154$

Similarly $23 \times 11 = 2\,(2 + 3)\,3 = 253$

$97 \times 11 = 9\,(9 + 7)\,7 = 1067$

$72 \times 11 = 7\,(7 + 2)\,2 = 792.$

Let us find how this pattern has been formed.

```
 763×11
 ------
  763
 763
 ------
 8393
 ------
```

In the product we note the following :

1. Digit in the unit's place :

   Digit in the unit's place (3) of the multiplicand.

2. Digit in the Ten's Place :

   Sum of the digits in the unit's and ten's place (3+6=9) of the multiplicand.

3. **Digit in the hundred's place :**

Sum of the digits in the 10's and 100's place $(6+7=13)$ of the multiplicand.

4. Digit in the thousand's place :

Add the carry over in the previous step to the hundred's place of the given number $(7+1=8)$.

Example 1 :

Let us consider 695×11.

In the product :

1. Digit in the unit's place : 5
2. Digit in the ten's place : $(5+9)=14$. Take only 4.
3. Digit in the hundred's place : $1+(9+6)=16$. Take only 6.
4. Digit in the thousand's place : $1+6=7$.

$$\begin{aligned} \text{Now,} \quad 695 \times 11 &= 6\,(6+9)\,(9+5)\,5 \\ &= 6\,(15)\,(14)\,5 \\ &= (6+1)\,(5+1)\,4\,5 \\ &= 7645. \end{aligned}$$

Example 2 :

$$\begin{aligned} & 9287 \times 11 \\ &= 9\,(9+2)\,(2+8)\,(8+7)\,7 \\ &= 9\,(11)\,(10)\,(15)\,7 \\ &= (9+1)\,(1+1)\,(0+1)\,5\,7 \\ &= 102157. \end{aligned}$$

EXERCISE 2-6 (Oral)

Find the product of :

1.	27×11	6.	413×11
2.	63×11	7.	984×11
3.	75×11	8.	1980×11
4.	89×11	9.	4895×11
5.	234×11	10.	9267×11

3–1. Squares

Consider the following products :

$1\times1=1$	$4\times4=16$	$7\times7=49$
$2\times2=4$	$5\times5=25$	$8\times8=64$
$3\times3=9$	$6\times6=36$	$9\times9=81$
		$10\times10=100$

The product of a number multiplied by itself is called the square of the number.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ……… are called square numbers.

Note the digits in the unit's place of these numbers.

They are 0, 1, 4, 5, 6, 9. We do not find 2, 3, 7, 8 in the unit's place of square numbers. Therefore we may say that the numbers ending with 2, 3, 7, 8 are not square numbers. But it may also be noted that all numbers ending with 0, 1, 4, 5, 6, 9 are not necessarily square numbers.

Example : 21, 14, 35, 6, 19 – are not square numbers.

If a number ends with 0, 1, 4, 5, 6, 9 it may be a square number or may not be. But if a number ends with 2, 3, 7 (or) 8 it is definitely not a square number.

More over, the square of odd numbers are also odd numbers and the square of even numbers are also even numbers. The squares of even numbers are divisible by 4.

EXERCISE 3–1 (a)

1. Find the squares of all the natural numbers upto 30.
2. Find the squares of the following numbers :

(1) 42	(4) 197	(7) 823	(10) 1239.
(2) 81	(5) 238	(8) 874	
(3) 90	(6) 625	(9) 926	

3. By just looking at the unit's digit, state which of the following are not square numbers for certain :

(1) 625	(4) 442	(7) 2698	(10) 90.
(2) 121	(5) 1843	(8) 40000	
(3) 196	(6) 169	(9) 4697	

3 1-1. Index Notation

We write 1×1 as 1^2

2×2 as 2^2

3×3 as 3^2

136×136 as 136^2.

2 is called the Index. It means the number has been multiplied by itself twice.

8^2 means 8×8.

8^2 is read as 8 squared.

Example :

$$3 \times 4 \times 7 \times 3 \times 7 \times 4 = (3\times3) \times (4\times4) \times (7\times7)$$
$$= 3^2 \times 4^2 \times 7^2.$$

EXERCISE 3-1 (b)

1. Rewrite the following using indices :

(1) 6×6 (2) 123×123 (3) $2 \times 2 \times 3 \times 3$

(4) $5 \times 5 \times 9 \times 9 \times 12 \times 12$ (5) $4\times9\times7\times4\times9\times7$

(6) $14 \times 11 \times 14 \times 15 \times 11 \times 15$.

2. Rewrite using indices :

(1) 25	(2) 49	(3) 169	(4) 400	(5) 36
(6) 196	(7) 225	(8) 900	(9) 144	(10) 1225.

3. Find the answer :

(1) 8^2 (2) 25^2 (3) 44^2 (4) $8^2 \times 25^2$

(5) $12^2 \times 15^2$ (6) $3^2 \times 5^2 \times 7^2$.

3-2. Cubes

Examine the following patterns:

$1 \times 1 \times 1 = 1$	$6 \times 6 \times 6 = 216$
$2 \times 2 \times 2 = 8$	$7 \times 7 \times 7 = 343$
$3 \times 3 \times 3 = 27$	$8 \times 8 \times 8 = 512$
$4 \times 4 \times 4 = 64$	$9 \times 9 \times 9 = 729$
$5 \times 5 \times 5 = 125$	$10 \times 10 \times 10 = 1000.$

The numbers 1, 8, 27, ... 1000 are called cubes. The cube of a number is got by multiplying the number by itself thrice.

EXERCISE 3-2 (a)

1. (a) What do you find in the unit's place of the cubes in the above examples?

   (b) Can you say whether a number is a cube or not at a glance?

   (c) State the pattern of the digits in the unit's place of the cubes of the numbers?

2. State the cubes of the following numbers.

   (a) 11 (b) 12 (c) 15 (d) 20 (e) 100.

3-2-1. Cubes – Exponential Method

$5 \times 5 \times 5$ can be written as 5^3 in exponential form. 5^3 should be read as 5 cubed.

EXERCISE 3-2 (b)

1. Rewrite using exponents:

   (a) $11 \times 11 \times 11$ (b) $17 \times 17 \times 17$

   (c) $3 \times 3 \times 3 \times 2 \times 2 \times 2$

   (d) $7 \times 7 \times 7 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

   (e) $9 \times 7 \times 5 \times 9 \times 7 \times 5 \times 9 \times 7 \times 5$

   (f) $2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 2 \times 3 \times 4$

2. Answer the following:

(a) 13^3 (b) 16^3 (c) 25^3 (d) 30^3 (e) 61^3.

3-3. Square Root

(a) Finding square root by the method of factorization :

If a number can be expressed as a product of two equal factors, that factor is called the square root of that number.

1. $9 = 3 \times 3$.

   Square of 3 is 9. $\therefore$ Square root of 9 is 3.

2. $5 \times 5 = 25$.

   Square of 5 is 25. Square root of 25 is 5.

3. $8 \times 8 = 64$.

   Square of 8 is ——

   Square root of —— is —— .

   Square root of 64 can be written as $\sqrt{64}$.

   The symbol for square root is " $\sqrt{}$ ".

Example 1 :

Find the square root of 441 by factorization.

$$\begin{array}{r|l} 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & \;\;7 \end{array}$$

$\therefore$ $441 = 3 \times 3 \times 7 \times 7 = 3^2 \times 7^2$

$\sqrt{441} = 3 \times 7 = 21$

Verify whether the square of 21 is 441.

Example 2 :

Find the square root of 225 by factorization.

$$\begin{array}{r|l} 5 & 225 \\ \hline 5 & 45 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$225 = 5 \times 5 \times 3 \times 3$$
$$= 5^2 \times 3^2$$
$$\sqrt{225} = 5 \times 3 = 15$$

$\therefore$ Square root of 225 is $5 \times 3 = 15$.

Example 3 :

$$\begin{array}{r|l} 2 & 484 \\ \hline 2 & 242 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

$\sqrt{484} = ?$

$$484 = 2 \times 2 \times 11 \times 11 = 2^2 \times 11^2$$

$\therefore \quad \sqrt{484} = 2 \times 11 = 22.$

We understand the following fact from the above examples. The square of 15 is 225 and the square root of 225 is 15. Therefore finding the square and the square root are inverse operations.

It can be figuratively mentioned like :

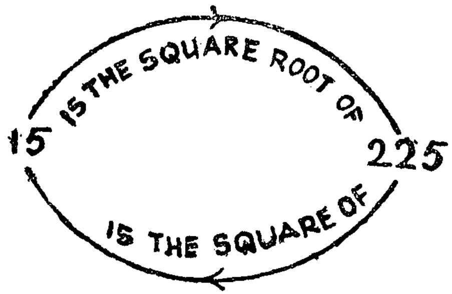


Fig. 1-1

EXERCISE 3–3

1. Find the square root of :

   (1) 121 (3) 225 (5) 400 (7) 1600
   (2) 144 (4) 256 (6) 900 (8) 10000.

2. Find the following :

   (1) $\sqrt{196}$ (3) $\sqrt{4900}$ (5) $\sqrt{6400}$.
   (2) $\sqrt{169}$ (4) $\sqrt{90000}$

3. Find the value of :

   (1) $\sqrt{16 \times 16}$ (4) $\sqrt{19^2 \times 2^2}$
   (2) $\sqrt{17^2}$ (5) $\sqrt{4^2 \times 3^2 \times 7^2}$
   (3) $\sqrt{18^2}$ (6) $\sqrt{7^2 \times 5^2 \times 4^2}$

4. Find the square root by factorisation :

   (1) 784 (4) 4225 (7) 1225 (10) 11025.
   (2) 1936 (5) 441 (8) 9025
   (3) 2025 (6) 576 (9) 5625

5. Find the value of :

   (1) $\sqrt{15 \times 21 \times 35}$ (4) $\sqrt{32 \times 8 \times 16}$
   (2) $\sqrt{6 \times 12 \times 8}$ (5) $\sqrt{15 \times 10 \times 3 \times 2}$
   (3) $\sqrt{48 \times 56 \times 42}$ (6) $\sqrt{28 \times 21 \times 4 \times 3}$

3-4. Play Way in Squares

Examine the following Pattern :

Square :	1	4	9	16	25	36	49	64	81
Square Root :	1	2	3	4	5	6	7	8	9
Digit in the unit's place of the Square :	1	4	9	6	5	6	9	4	1

The square root of the numbers having 1 in their units place end with 1 and 9. In the same manner square root of the numbers having 6 in their units place end with 4 and 6.

$$1 + 9 = 10\,; \quad 4 + 6 = 10.$$

Don't you know that 1 and 9 and 4 and 6 are complements of 10 ?

HISTORICAL NOTE

In the word 'square root' the first letter of the word 'root' is *r*. This has become "$\sqrt{}$" in due course. The origin of this sign is traced to a book written by a German called Rudolff as early as 1525 A. D. But it was written without the overhead line.

Finding square root in a flash

Example :

$15 \times 15 = 225$	$[2 = 1 \times 2]$
$25 \times 25 = 625$	$[6 = 2 \times 3]$
$35 \times 35 = 1225$	$[12 = 3 \times 4]$
$45 \times 45 = 2025$	$[20 = 4 \times 5]$
$125 \times 125 = 15625$	$[156 = 12 \times 13]$

The numbers which you find in the tens and units place are 2 and 5 respectively.

Note :

This method is applicable to those numbers which end with 5.

Square Root—Ancient Chinese Method

The Chinese, in ancient times discovered that continuous sums of odd natural numbers give square numbers.

For example,

$$1$$
$$1 + 3 = 4$$
$$1 + 3 + 5 = 9$$
$$1 + 3 + 5 + 7 = 16$$
$$1 + 3 + 5 + 7 + 9 = 25$$

....

....

Therefore to find the square root of a square number, the odd numbers from 1 onwards can be subtracted from the number till the remainder 0 is obtained.

Example :

$\sqrt{36} = ?$

$$\begin{array}{rl} 36 & \\ -\ 1 & (1) \\ \hline 35 & \\ -\ 3 & (2) \\ \hline 32 & \\ -\ 5 & (3) \\ \hline 27 & \\ -\ 7 & (4) \\ \hline 20 & \\ -\ 9 & (5) \\ \hline 11 & \\ -11 & (6) \\ \hline 0 & \\ \hline \end{array}$$

First six odd natural numbers have been subtracted here from 36 to get 0 as remainder. The odd natural number 11 is subtracted for the 6th time. Hence square root is

$$\frac{1 + 11}{2} = 6.$$

$$\therefore \quad \sqrt{36} = 6.$$

Similarly use the above method to find the square root of 100, 225, etc. and verify.

4—1. Greatest Common Divisor (g. c. d.), Least Common Multiple (l. c. m.) :

PRE—TEST

Exercise 4—1

1. Find the g. c. d. of :
   (1) 12, 16 (2) 12, 15 (3) 20, 25 (4) 96, 112.
2. Find the l. c. m. of :
   (1) 3, 4 (2) 4, 6 (3) 25, 30 (4) 75, 90.

4—2. Greatest Common Divisor (Three Numbers) :

We have already learnt how to find the g. c. d. of two natural numbers. Let us now try to find the g. c. d. of 3 natural numbers.

Example 1 :

Find the g. c. d. of 6, 8 and 10.

First find the g. c. d. of 6, 8. Then the g. c. d. of this number and 10 may be found. Now we get the g. c. d. of 6, 8 and 10.

Step (1) : g. c. d. of 6, 8.

Express 6 and 8 as the product of prime factors.

$6 = 3 \times 2$; $8 = 2 \times 2 \times 2 = 2^3$.

g. c. d. = 2.

Step (2) : We shall now find the g. c. d. of 2 and 10.

$2 = 2$

$10 = 2 \times 5$

g. c. d. = 2

$\therefore$ g. c. d. of 6, 8, 10 is 2.

This can be shown as follows :

$6 = 2 \times 3 \qquad = 2 \times 3$

$8 = 2 \times 2 \times 2 \quad = 2^3$

$10 = 2 \times 5 \qquad = 2 \times 5$

$\therefore$ g. c. d. = 2

Example 2:

Find the g. c. d. of 48, 72 and 60

(1) g. c. d. of 48, 72

$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

$\therefore$ g. c. d. of 48, 72 is $2^3 \times 3 = 24$

(2) Now let us find the g. c. d. of 24, 60

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

$\therefore$ g. c. d. of 24, 60 is $2^2 \times 3 = 12$

$\therefore$ g. c. d. of 48, 72 and 60 is 12

This can be shown as follows :

$48 = 2^4 \times 3$

$72 = 2^3 \times 3^2$

$60 = 2^2 \times 3 \times 5$

$\therefore$ g. c. d. $= 2^2 \times 3 = 12$

Example 3 :

Ribbons of lengths 12 m, 10 m, and 14 m are cut into bits of equal lengths. What is the maximum length of a bit ?

We have to find the g.c. d. of 12, 10, 14.

$12 = 2^2 \times 3$

$10 = 2 \times 5$

$14 = 2 \times 7$

$\therefore$ g. c. d. = 2.

$\therefore$ The maximum length of a bit is 2 m.

Exercise 4—2

1. Find the g. c. d. of : (oral)

   (1) 6, 9, 12. (2) 8, 12, 20 (3) 16, 32, 64
   (4) 30, 60, 45

2. Find the g. c. d. of :

   (1) 175, 125, 150 (2) 70, 154, 126 (3) 66, 165, 264
   (4) 91. 208, 65

3. The strength of 6th, 7th and 8th standards of a school are 200, 160 and 120 respectively. The students of each

standard are to be divided into squads, each containing equal number of students and the squad should be as big as possible. Find the maximum number of students in each squad ?

4. The number of composition note books in three bundles are 96, 144 and 120. If they are to be arranged in equal bundles without mixing them up, what is the maximum number of note books to be put in each bundle ?

4—3. Least Common Multiple (Three Numbers) :

We have learnt in the previous class how to find the l.c.m. of two natural numbers. Let us now learn how to find the l. c. m. of three natural numbers.

Example :

Find the l. c. m. of 30, 20 and 40.

Method 1

First find the l. c. m. of 30 and 20.

The multiples of 30 are : 30, 60, 90, 120, ...

The multiples of 20 are : 20, 40, 60, 100, 120, ...

The common multiples are 60, 120, ...

The smallest of these is 60.

∴ l. c. m. of 30 and 60 is 60.

Then find the l. c. m. of 60 and 40.

The multiples of 60 are : 60, 120, 180, 240, ...

The multiples of 40 are : 40, 80, 120, 160, 200, 240, ...

The common multiples are : 120, 240, ...

∴ The l. c. m. of 60 and 40 is 120.

The l. c. m. of 30, 20 and 40 is 120.

Method 2

By prime factorization the l. c. m. can now be found out.

$30 = 2 \times 5 \times 3$

$20 = 2^2 \times 5$

$40 = 2^3 \times 5$

l. c. m. $= 2^3 \times 5 \times 3 = 120.$

Look at the following procedure carefully :

Step	Divisor	Numbers
Common divisor for 20, 30 and 40 is 2.	2	20, 30, 40
Common divisor for 10, 15 and 20 is 5.	5	10, 15, 20
Common divisor for 2 and 4 is 2.	2	2, 3, 4
No common divisor except 1		1, 3, 2

$\therefore$ l. c. m. $= 2 \times 5 \times 2 \times 1 \times 3 \times 2 = 120.$

Note :

$30 = 2 \times 5 \times 3$

$20 = 2^2 \times 5$

$40 = 2^3 \times 5$

g. c. d. $= 2 \times 5 = 10$

l. c. m. $= 2^3 \times 5 \times 3 = 120$

For g. c. d. we take the least of the exponents. For l. c. m. we take the highest of the exponents.

Exercise 4 -3

1. Find the l. c. m. of : (oral)

   (1) 2, 4, 6 (2) 3, 6, 9 (3) 15, 20, 10 (4) 20, 50, 25.

2. Find the l. c. m. of :

   (1) 135, 60, 120 (2) 72, 108, 120 (3) 208, 91, 117 (4) 91, 70, 35.

3. A copper wire can be cut into lengths of 18 cm or 12 cm or 24 cm. What is the smallest length of the copper wire that can be taken up for this purpose ?

4. A cart load of oranges can be put into baskets each holding 48 or 54 or 72 oranges respectively. If the number of oranges in each basket is to be equal, find the minimum number of oranges that must be in the cart ?

5. What is the smallest number which gives the remainder 1 when divided by 2, 3, 4, 5 and 6 ?

6. What is the smallest number which gives the remainder 9, 7, 5, 3 when divided by 10, 8, 6 and 4 respectively ?

7. What is the smallest number which gives the remainder 1, 2, 3, 4 and 5 when divided by 2, 3, 4, 5 and 6 respectively ?

8. Find the smallest number when divided by 10, 16 and 25 yields the remainders 3, 9 and 18 respectively.

TEST PAPER 1

[Time 1 hour]

1. (1) Find the place value of the numbers in thick type.
 9 8 7 **9** 4 9.

 (2) Round off 5 4 9 2 8 7 correct to ten thousand.

 (3) Add by short method :
 4282 + 9997.

 (4) Fill in the blanks suitably :
 To multiply a number by 25, it must be multiplied by ——— and then divided by ——— .

 (5) Find by short method :
 246×15

 (6) Find the g. c. d. of : 5, 10, 15.

 (7) Find : $3^2 \times 5^2$.

 (8) Rewrite using Exponents :
 $7 \times 7 \times 6 \times 5 \times 6 \times 5 \times 7 \times 5 \times 6$.

 (9) $\sqrt{225} = ?$

 (10) Find the l. c. m. of : 6, 12, 18.

2. (1) Add using short method :
$86 + 35 + 41$.

(2) Multiply by short method :
(a) $84 \times 15 \times 5$ (b) 2349×11

(3) Find the g. c. d. and l. c. m. of
(a) 150, 180, 75 (b) 32, 128, 64

(4) The number of Mathematics books in three bundles are 96, 64 and 80. Without mixing them rearrange and pack them into bundles each containing equal number of books. Find the maximum number of books in each bundle.

(5) A cartload of mangoes can be put into baskets, so that each may contain 72 or 108 or 144 fruits. If the number of mangoes in each basket is to be equal find the maximum number of mangoes that must have been in the cart ?

3. (1) Find the value of : $4^2 \times 5^2$.

(2) Find if 27598 is a square number. State the reason.

(3) Find the square root by factorization
$45 \times 27 \times 15$.

(4) Find the g. c. d. of
$2^3 \times 3 \times 5$, $2^2 \times 3^2 \times 5$, $2 \times 3^3 \times 5^2$.

(5) Find the value of : $\sqrt{5^2 \times 3^2 \times 2^2}$

4. (1) Find the square root by factorization : 11025.

(2) Find the value of : $\sqrt{91 \times 195 \times 105}$

(3) The side of a square field is 135 m. Find its area.

(4) The area of a square field is 7056 Sq. m. What is the length of its side ?

(5) Balan exchanged a rectangular plot of ground of length 132 m and breadth 33 m for a square ground of equal area. Find the side of the square field.

KAPREKAR NUMBER

Take a four digit number, which has all the four digits different. Arrange the digits in descending order and write the number. Again arrange the digits in ascending order and write the number. Find the difference of the two numbers. We have now found the successor of the number taken. Repeat this process as shown in the example.

Example :

	Number taken	6587	
I.	Descending order	8765	
	Ascending order	5678	
	Difference	3087	Successor of 6587
II.	Descending order	8730	
	Ascending order	0378	
	Difference	8352	Successor of 3087
III.	Descending order	8532	
	Ascending order	2358	
	Difference	6174	Successor of 8352
IV.	Descending order	7641	
	Ascending order	1467	
	Difference	6174	Successor of 6174

In the III step we get the successor as 6174 and after this we are not able to get any new successor.

Take any other four digit number and play with it as above. Find in which step you get 6174. Come on! Start!

This 6174 is called Kaprekar Number. Mr. D. R. Kaprekar is an Indian mathematician. All numbers are his friends. He has invented several number games.

Let us now see another number game of his.

Take a three digit number and let us find its successor in a new way. Look at the example.

1. 237 Number
2. 519 [2 + 3; 3 + 7 = 10, 1 + 0; 7 + 2]
3. 615 [5 + 1; 1 + 9 = 10, 1 + 0; 9 + 5 = 14, 1 + 4]
4. 762 [6 + 1; 1 + 5; 5 + 6 = 11, 1 + 1]
5. 489 [7 + 6 = 13, 1 + 3; 6 + 2; 2 + 7]
6. 384 [4 + 8 = 12, 1 + 2; 8 + 9 = 17, 1 + 7; 9 + 4 = 13, 1 + 3]
7. 237 [3 + 8 = 11, 1 + 1; 8 + 4 = 12, 1 + 2; 4 + 3]

We get the original number 237 again in the 7th step.

Can you find out the number which comes again in the next step itself.

Play with numbers. Make them your friends.

ANSWERS

Exercise 1—1

(1) 1 (2) 0 (3) yes; 0 (4) Hundred

(5) Two crore seventy four lakh thirty five thousand six hundred eighteen.

(6) One million nine hundred eleven thousand four hundred ten.

(7) 1 09 82 109 (8) 7 608 908

(9) $8 \times 10000 + 1 \times 1000 + 9 \times 10 + 9$

(10) $4 \times 10^4 + 6 \times 10^3 + 3 \times 10^1 + 4$

(11) 97 06 029 (12) 3952, 3592, 3529, 3295, 3259

(13) 27 105 (14) 53 (15) 9 28 854 (16) 1 09 023

(17) 51 hundreds (18) 647.

Exercise 1—4

(1) 87 (2) 162 (3) 193 (4) 166 (5) 171
(6) 146 (7) 170 (8) 260 (9) 280 (10) 320

Exercise 2—3

(1) 165 (2) 345 (3) 525 (4) 705 (5) 885
(6) 1365 (7) 1545 (8) 3225 (9) 4905 (10) 6585

Exercise 2—4

(1) 40920 (2) 28050 (3) 62370 (4) 5700
(5) 48510 (6) 15050 (7) 52920 (8) 73920
(9) 143520 (10) 138600

Exercise 3—1 (a)

1. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144
169, 196, 225, 256, 289, 324, 361, 400, 441
484, 529, 576, 625, 676, 729, 784, 841, 900.

2. (1) 1764 (2) 6561 (3) 8100 (4) 38809
(5) 56644 (6) 390625 (7) 677329 (8) 763876
(9) 857476 (10) 1535121.

3. (4) 442 (5) 1843 (7) 2598 (9) 4697 are not square numbers for certain.

Exercise 3—1 (b)

1. (1) 6^2 (2) 123^2 (3) $2^2 \times 3^2$ (4) $5^2 \times 9^2 \times 12^2$
(5) $4^2 \times 7^2 \times 9^2$ (6) $11^2 \times 14^2 \times 15^2$.

2. (1) 5^2 (2) 7^2 (3) 13^2 (4) 20^2 (5) 6^2 (6) 14^2
(7) 15^2 (8) 30^2 (9) 12^2 (10) 35^2.

3. (1) 64 (2) 625 (3) 1936 (4) 40000 (5) 32400
(6) 11025.

Exercise 3—2 (a)

1. (a) The digits obtained are from 0 to 9.
   (b) Impossible.
   (c) 2, 3, 5, 7, 8 – The sum of each number and the units digit of its cube is equal to 10.

2. (a) 1331 (b) 1728 (c) 3375 (d) 8000 (e) 10 00 000.

Exercise 3 – 2 (b)

1. (a) 11^3 (b) 17^3 (c) $3^3 \times 2^3$ (d) $7^3 \times 2^3 \times 5^3$ (e) $5^3 \times 7^3 \times 9^3$ (f) $2^3 \times 3^3 \times 4^3$

2. (a) 2197 (b) 4096 (c) 15625 (d) 27000 (e) 226981

Exercise 3—3

1. (1) 11 (2) 12 (3) 15 (4) 16 (5) 20 (6) 30 (7) 40 (8) 100

2. (1) 14 (2) 13 (3) 70 (4) 300 (5) 80

3. (1) 16 (2) 17 (3) 18 (4) 38 (5) 84 (6) 140

4. (1) 28 (2) 44 (3) 45 (4) 65 (5) 21 (6) 24 (7) 35 (8) 95 (9) 75 (10) 105

5. (1) 105 (2) 24 (3) 336 (4) 64 (5) 30 (6) 84

Exercise 4—2

2. (1) 25 (2) 14 (3) 33 (4) 13
3. 40 4. 24

Exercise 4—3

2. (1) 1080 (2) 1080 (3) 13104 (4) 910
3. 72 cm 4. 432 5. 61
6. 119 7. 59 8. 393

Test Paper 1

1. (1) 9 Hundreds (2) 550000 (3) 14279

   (4) 100, 4 (5) 3690 (6) 5 (7) 225

   (8) $5^3 \times 6^3 \times 7^3$ (9) 15 (10) 36

2. (1) 165 (2) (i) 6300 (ii) 25839

   (3) (1) 15; 990 (2) 32; 128

   (4) 16 (5) 432

3. (1) 2000 (2) No. The number ends with 8. (3) 135

   (4) 30 (5) 30

4. (1) 105 (2) 1365 (3) 15625 sq. m. (4) 84 m.

   (5) 66 m.

2. INTEGERS

In the earlier standard, we have seen that to divide 2 by 3 we needed to extend the number system and formed a system of fractional numbers. Then we also found that it is not possible to subtract 3 from 2 in the system of whole numbers. Let us now see how this inadequacy in the set of whole numbers may be removed.

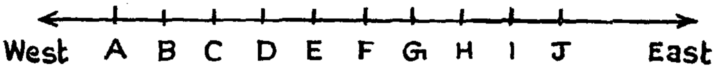


Fig. 2—1.

Fig. 2—1 shows various places like college, bank, etc., on a straight road running from West to East. Each place is located 1 km apart from its neighbouring place. Let A, B, C, ... denote these places as under.

A - College, B - Bank, C - Post Office, D - Market, E - Higher Secondary School, F - Church, G - Mariamman Temble, H - Elementary School.

Kari and Mari start from D and travel 2 km. Can we say where they will be ?

Do we get the same answer for all ?

Some may answer as B and others may answer as F. We can say definitely only if we know the direction in which they moved 2 km from D.

It Mari and Kari start from D, Mari moving 2 km towards east and Kari moving 2 km towards west, then we can definitely say that Mari will be at F and Kari will be at B respectively.

It is therefore necessary to know not only the distance travelled, but also the direction of journey. We may attach a directional sign to each number to denote these two factors. Such numbers which are associated with directions are called **directed numbers.**

Instead of saying that the road is running from West to East we can mark the distances of the places by arrow heads as shown in the figure to show the direction.

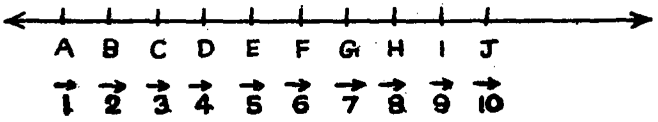


Fig. 2—2.

Let us now take D as the starting point and mark the distances of the places. A is to the west of D and is 3 km from D. So we put $\overleftarrow{3}$ at A. Similarly, since H is 4 km east of D, we put $\overrightarrow{4}$ at H. Such numbers which have magnitude and direction are called ' direct numbers'.

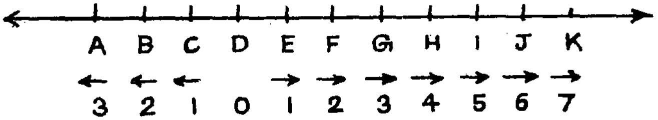


Fig. 2—3.

Since D is the starting point it is neither east nor west of D. No arrow head is necessary. Like directions (East, West), we may also represent situations such as (Profit, loss), (Up, down), (Increase, decrease), (Remittance, withdrawals in a bank), (Left, Right), etc., ... as directed numbers.

Exercise 1—1 (Oral)

1. Represent the following by arrow heads :

   (a) 800 m above the mean sea level ; 400 m below the mean sea level.

   (b) Profit Rs. 80 ; Loss Rs. 20.

   (c) A sum of Rs. 500 is remitted into a bank ; Rs. 300 is withdrawn,

   (d) I went 50 km towards left and then 20 km towards right.

2. Directed Numbers—Representation Along the Number Ray

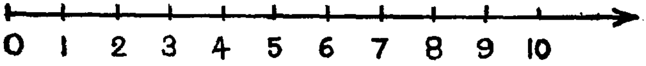


Fig. 2—4.

In the number line. you may note that numbers more than 5 are to the right of 5 and numbers less than 5 are to its left. Let us take 5 as the starting point and mark the other points in the number line as directed numbers.

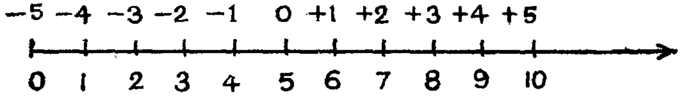


Fig. 2—5.

5−1= 4 Hence 4 is represented as −1.

5−2= 3 Hence 3 is represented as −2

5−3= 2 Hence 2 is represented as −3.

5−4= 1 Hence 1 is represented as −4.

5−5= 0 Hence 0 is represented as −5.

5+5=10 Hence 10 is represented as +5.

5+4= 9 Hence 9 is represented as +4.

5+3= 8 Hence 8 is represented as +3.

5+2= 7 Hence 7 is represented as +2.

5+1= 6 Hence 6 is represented as +1.

5+0= 5 Hence 5 is represented as 0.

Mathematicians therefore represent directed numbers by using '+' or '—' sign with the whole numbers. We may see that 0 does not have any directed sign since 5+0=5 and 5—0=5 and it can take any of the two signs.

3. Positive, Negative Numbers and Zero :

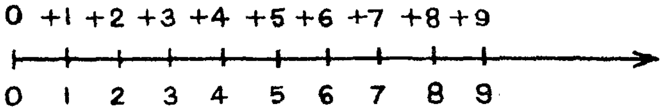


Fig. 2—6.

If the number ray starts from zero (0) instead of starting from 5 it will be represented as above. We find all the natural numbers taking '+' sign. All numbers to the left of 0 in the number line will therefore take '—' sign.

Numbers with '+' sign are called positive numbers, and numbers with '—' sign are called negative numbers. All natural numbers are therefore positive numbers.

We may now draw the extended number line, consisting of positive numbers, negative numbers and zero.

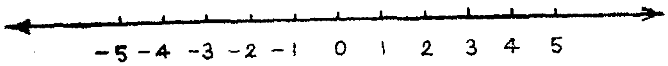


Fig. 2—7.

0 is neither positive nor negative. *Positive numbers,* negative numbers and zero form the system of Integers.

Denoting numbers with '+' or '−' sign is the standard numeral for integers. Thus +3, +4, −3, −5, 0 are the standard numerals for some of the integers.

4. Order in Number Line :

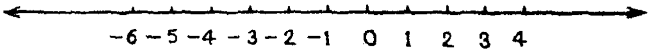


Fig. 2—8.

Look at the number line

5 is to the right of 2. ∴ $5 > 2$

2 is to the left of 5. ∴ $2 < 5$.

−2 is to the left of 0. ∴ $-2 < 0$.

−3 is to the right of −4. So $-3 > -4$.

−8 is to the left of −3 So $-8 < -3$.

In the number line numbers to the right of a number are greater than *that number and numbers to its left* are less than that number. We therefore find that numbers increase as we move to the right and decrease as we move to the left in the number line.

Example 1 :

Write in ascending order :

−2, −4, −5, −1, 0, 2, 4

Let us represent these in the number line.

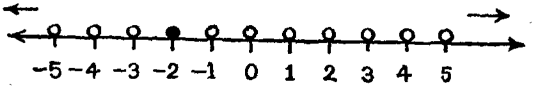


Fig. 2—9.

The number on the left extreme is -5.

-4 is to its immediate right, $-4 > -5$.

-2 is to the right of -4; $-2 > -4$

-1 is to the right of -2; $-1 > -2$.

0 is to the right of -1; $0 > -1$.

2 is to the right of 0; $2 > 0$.

4 is to the right of 2; $4 > 2$.

So the above numbers if arranged in ascending order will be $-5, -4, -2, -1, 0, 2, 4$. With the help of inequality sign $<$, they may be denoted as

$$-5 < -4 < -2 < -1 < 0 < 2 < 4.$$

Example 2 :

Write in descending order : $-6, 2, 0, -5, 3, -1, 4$. Let us denote these numbers on a number line.

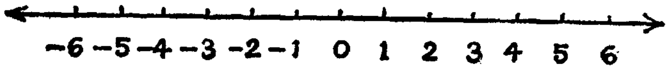


Fig. 2—10.

These integers can be written in descending order while moving from right to left, the biggest number coming first and the smallest number coming last.

$$\therefore \; 4 > 3 > 2 > 0 > -1 > -5 > -6.$$

Exercise 4—1

1. Put the appropriate sign '>' or '<' in the dotted space and read.

(1) -1 1   (4) -8 8

(2) -3 3   (5) -6 4

(3) 10 2   (6) -1 ...... 0.

2. Express the *relationship between the numbers* using '$>$' :

(1) 200. 300 (4) $-14, -16$

(2) -14, 20 (5) 0, -2

(3) 299, -399 (6) -5, 0.

3. Express the relationship between the numbers using '$<$' :

(1) 0, 5 (3) -1980, -2010

(2) -5, 5 (4) 510, -410.

4. Write in ascending order :

-2, 2, 3, -13, 6, 0, -8, 10.

5. Write in descending order :

11, -11, 6, -6, 0, 2, -1, -4.

6. Between -5 and 4,

(a) How many integers are there ?

(b) How many whole numbers are there ?

(c) How many natural numbers are there ?

5—1. Addition in Integers (a)

We have learnt how *to add the* whole numbers in a number ray. Now we will see *how to add the* **integers in a** number line.

As in the case of whole numbers, we start from **0 in the** number line to add two or three numbers. The distance we have moved from 0 gives us the sum of the integers.

The difference between the *addition in* whole numbers and in integers is that we move always to the *right* ***in the whole***

number system whereas in integers for + 3, we move to the right three places and for —3, we move to the left three places.

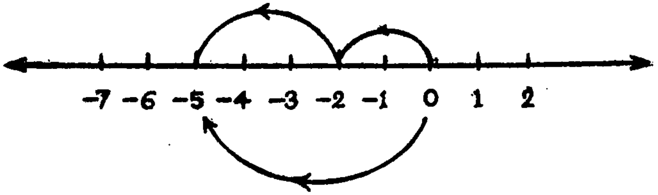


Fig. 2—11.

To find (—2) + (—3), we start from 0, move a distance of 2 units to the left and again move another 3 units to the left. We will be 5 units to the left of 0. Fig. 2-11 shows this fact.

$$(-2) + (-3) = (-5).$$

Exercise 5—1

Find the sum in the number line :

(a) (—3) + (—4) (d) (—6) + (—4)

(b) (—5) + (—2) (e) (—8) + (—3)

(c) (—4) + (—2)

5—2. Addition in Integers (b)

Example :

Find the value of 5 + (—2) ·

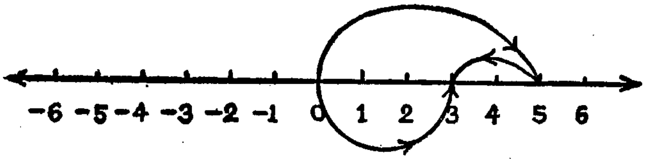


Fig. 2—12.

Starting from 0, we move first 5 units to the right and then 2 units to the left. Now we will be at 3 units to the right of 0. Fig. 2-12 shows this fact.

$\therefore \quad 5 + (-2) = 3.$

Addition facts in Fig. 2-11 and Fig. 2-12 may also be shown as follows :

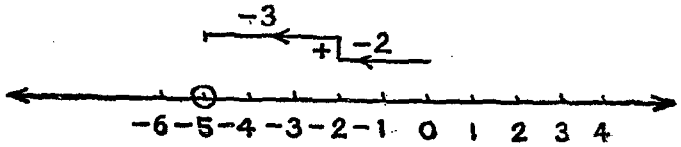


Fig. 2—13.

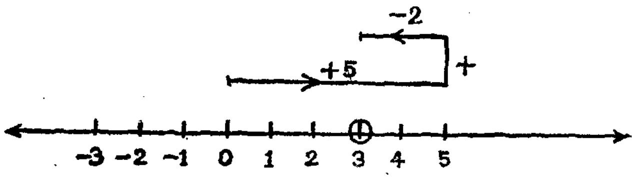


Fig. 2—14.

Exercise 5—2

Find the sum of the following in the number line.

(a) $8 + (-3)$ (b) $6 + (-3)$

(c) $4 + (-1)$ (d) $9 + (-8)$

(e) $10 + (-4)$

5—3. Addition in Integers (c)

Example :

Find the value of $2 + (-6)$.

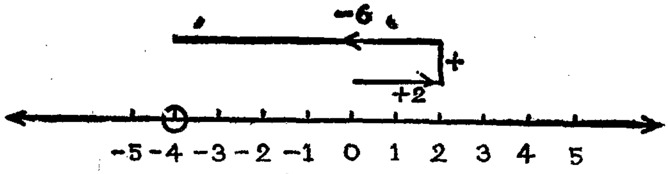


Fig. 2-15.

To find $2 + (-6)$, we move 2 units from 0 to the right and then move 6 units from there to the left. Now we will be at -4.

$\therefore \quad 2 + (-6) = -4.$

Exercise 5—3

Find the sum in the number line.

(a) $3 + (-6)$ (b) $4 + (-3)$

(c) $2 + (-7)$ (d) $5 + (-9)$

(e) $6 + (-10)$

5—4. Integers—Inverse Numbers

Example :

Find the value of $5 + (-5)$.

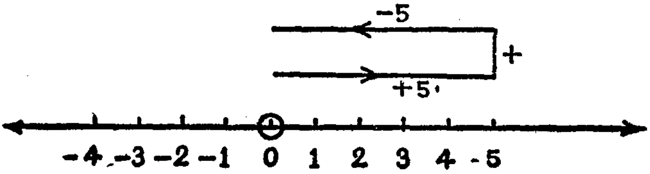


Fig. 2—16.

We start from 0, move 5 units to the right and then move 5 units to the left. Now we are back at 0.

$5 + (-5) = 0.$

-5 is called the additive inverse of 5.

Similarly we may show $(-5) + 5 = 0$. 5 is the additive inverse of -5.

Since $5 + 0 = 0 + 5 = 5$, 0 is called the additive identity, that is, a number that when added to another number, does not change the value of the other number.

Exercise 5—4

1. Find the sum of the following in a number line.

   (a) $6 + (-6)$ (b) $4 + (-4)$ (c) $7 + (-7)$

   (d) $3 + (-3)$ (e) $8 + (-8)$

2. Write the inverse of :

   (a) 6 (b) -4 (c) -7 (d) 3 (e) 8 (f) -10

   (g) 9 (h) 0.

5—5. Addition In Integers—The Property of Rearrangement

Examine carefully the following patterns in addition.

Pattern 1 :

(1)			(2)	(3)		
4	4	4	4	4	4	4
3	2	1	0	-1	-2	-3
7	6	5	4	3	2	1

(4)	(5)	
4	4	4
-4	-5	-6
0	-1	-2

What do we observe in the above patterns ?

1. The sum of any two positive integers is always a positive integers. (Example 1)

2. When the additive identity 0 is added to any integer, the result will be the same integer. (Example 2)

3 When a positive and a negative integer are added,

(a) A positive integer is obtained. (Example 3)

(b) A negative integer is obtained. (Example 5)

(c) Zero is got. (Example 4)

We further note the following.

1. The first addend remains the same.
2. The second addend decreases one by one.
3. The sum also decreases one by one.

Pattern 2:

(1)	4	3	(2)	2	(3)	1	(4)	0	(5)	−1	−2	−3
	−2	−2		−2		−2		−2		−2	−2	−2
	2	1		0		−1		−2		−3	−4	−5

What do we find in the above pattern ?

1. When a positive integer and a negative integer are added,

   (a) We get a positive integer (Example 1)

   (b) We get a negative integer (Example 3)

   (c) We get a zero. (Example 2)

2. When two negative integers are added the sum will be a negative integer. (Example 5)

3. When a negative integer is added to the additive identity 0, we get the same negative integer. (Example 4)

We note the following in the above patterns:

(a) One of the addends, that is −2, remains the same.

(b) The other addend decreases one by one.

(c) The sum also decreases one by one.

From the above two patterns we get the following facts.

1. The sum of two positive integers is a positive integer.
2. The sum of two negative integers is a negative integer.
3. Zero is an additive identity.

4. The sum of an integer and its inverse is 0. We get additive identity.

5. To find the sum of a positive integer and a negative integer

   (a) Subtract the smaller from the greater without considering signs.

   (b) Place the sign of the greater number to the sum.

Example 1 : $6 + (-8)$

Step 1. Here take -8 as 8.

2. On subtracting 6 from 8, the remainder is 2.

3. The sign of 8 is '$-$'.

4. $\therefore \; 6 + (-8) = -2$.

Example 2 : $(-5) + 9$

Here 9 is greater.

1. Treat -5 as 5.
2. Subtract 5 from 9. 4 is got.
3. The sign of 9 is '$+$'.
4. $\therefore \; (-5) + 9 = +4$.

Exercise 5—5

1. Find the sum of the following additive pairs using the number line :

(1) 6, 2 (2) -2, 4 (3) $7 - 3$

(4) 4, -8 (5) -9, 4 (6) -5, -5

(7) -2, -4 (8) 4, -4

2. Answer the following without the use of number line :

(1) $4 + (-10)$ (2) $(-10) + 16$

(3) $(-16) + (-12)$ (4) $15 + 20$

(5) $(-20) + (-80)$ (6) $60 + (-60)$

(7) $(-60) + (+60)$ (8) $(-20) + 4$

(9) $32 + (-62)$ (10) $(-27) + (-23)$

Fill up the following Addition Table.

SECOND NUMBER

FIRST NUMBER

+	-3	-2	-1	0	1	2	3	4
4								
3								
2								
1								
0								
-1								
-2								
-3								

(a) What are the numbers opposite to zero in the row of first numbers ?

(b) What are the numbers opposite to zero in the column of second numbers ?

(c) Write the pairs of integers whose sum is zero.

(d) Write the pairs of integers whose sum is – 2.

Look at the following diagrams and write the addition facts.

(i)

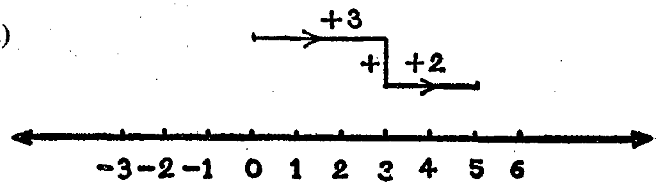


Fig. 2—17.

(ii)

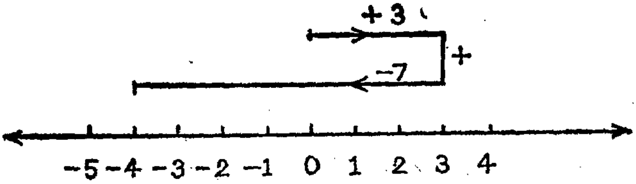


Fig. 2—18.

(iii)

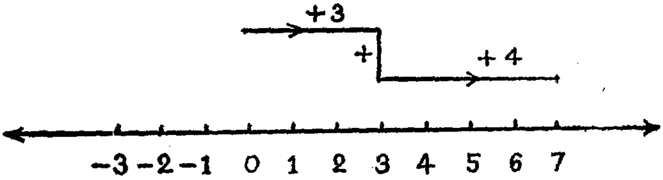


Fig. 2—19.

(iv)

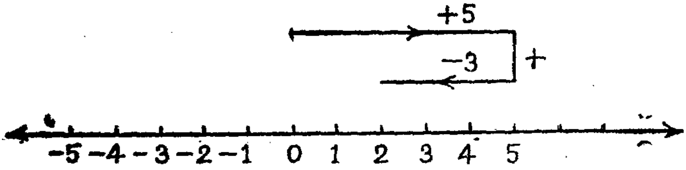


Fig. 2—20.

(v)

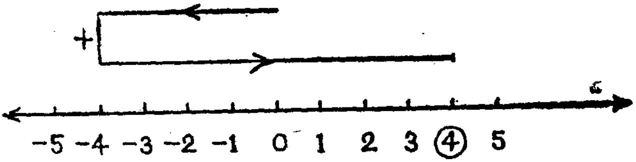


Fig. 2—21.

(vi)

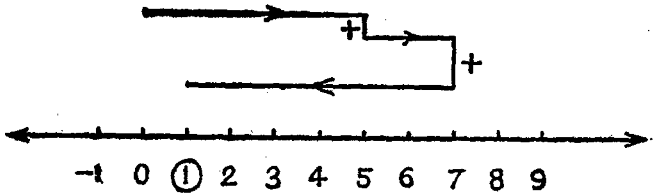


Fig. 2–22.

5–6. Subtraction in Integers

While studying about the whole numbers, we looked at the subtraction $8 - 3$ in two ways.

1. What is the remainder when 3 is subtracted from 8?
   $8 - 3 = \square$

2. What should be added to 3 to get 8?
   $3 + \square = 8.$

The subtraction in integers may similarly be looked at.

Example 1 :

Find the value of $4 - (-2)$.

" $4 - (-2)$ " is the same as " What should be added to -2 to get $+4$?"

$-2 + \square = 4$

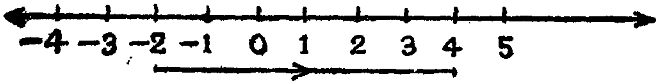


Fig. 2–23.

Starting from 0, we move to -2, take some steps and reach 4. How many steps we should take from -2 to reach 4? From the number line, we have to move 6 units to the right to reach 4.

We get $(-2) + 6 = 4$

Therefore $4 - (-2) = 6 = 4 + 2$

Example 2 :

Find $(-2) - 3$.

This is the same as to find the blank in $3 + \square = -2$.

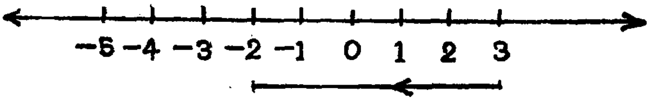


Fig. 2—24.

From 0 we first move to 3 and then we have to reach −2. For that, we have to move 5 units to the left.

Therefore,

$3 + (-5) = -2$.

or $(-2) - 3 = -5 = (-2) + (-3)$

Example 3 :

Find $3 - (-2)$.

We have to find the blank in $(-2) + \square = 3$.

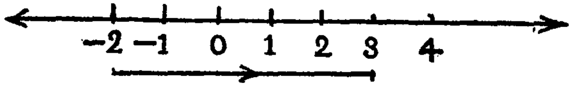


Fig. 2—25.

We have to reach 3 from −2 and for that we have to move 5 units to the right from −2.

Therefore,

$(-2) + 5 = 3$

or $3 - (-2) = 5 = 3 + 2$

Moreover we learn

$0 + (-5) = -5$

$0 - (+5) = -5$

$\therefore$ -5 can be written as $+(-5)$ or $-(+5)$

From the above, we find the following :

1. $4-(-2)=6=4+2$ (2 is the additive inverse of -2)
2. $-2-3=-5=-2+(-3)$ (-3 is the additive inverse of 3)
3. $3-(-2)=5=3+(+2)$ (2 is the additive inverse of -2)

Hence we understand that in the set of integers, subtraction can be easily transformed into addition. Snbtracting an integer is the same as adding its inverse.

Example :

Find $(-5)-(-7)=?$

Additive inverse of -7 is 7.

$\therefore\ (-5)-(-7)=(-5)+(+7)=2$

Exercise 5–6

1. Find the result using a number line :

(1) $5-2$ (2) $5-7$

(3) $(-5)-2$ (4) $(-6)-8$

(5) $(-4)-(-8)$ (6) $(-8)-(-2)$

(7) $6-(-8)$ (8) $5-(-2)$

2. Subtract :

(1) $232-137$ (2) $125-437$

(3) $(-627)-400$ (4) $(-200)-100$

(5) $(-200)-(-300)$ (6) $-150-(-50)$

(7) $200-(-100)$ (8) $350-(-250)$

6. Meaning of the Sign '–', '+'.

Look at the following carefully :

1. $(+4)\oplus(+6)$
   (a) Here the sign '+' in the circle represents the operation 'addition'.
   (b) In $+6$, '+' sign tells us that 6 is a positive number.

2. (a) 10 − 4. Here '−' sign represents the operation 'subtraction'.

   (b) 10 − (−3). In −3, '−' sign denotes the negative integer.

   (c) 4 + (−4) = 0. Here it represents that inverse of 4 is −4.

   (d) Inverse of −3 is − (−3) i.e. 3.

Hence, '−' sign, is used to represent

1. the operation 'subtraction'.
2. a negative integer.
3. the inverse of the other.

Exercise 6—1

1. What does the '+' sign within the circle mean in each of the following ?

(1) 8 ⊕ 2	(2) 9 ⊕ 2
(3) (+9) + (⊕8)	(4) (⊕2) + (+3)
(5) (+3) ⊕ (+4)	(6) (−4) ⊕ (−2)
(7) (−2) + (⊕3)	(8) (⊕5) + (−2)

2. What does the '−' sign within the circle mean in each of the following ?

(1) 9 ⊖ 2	(2) 8 ⊖ (−2)
(3) (−7) ⊖ (−8)	(4) 5 − (⊖2)
(5) (⊖7) − (3)	(6) (−5) − (⊖3)

3. Write the inverse of each of the following integers :

(1) −2	(2) 4	(3) 5	(4) −7	(5) 0
(6) 8	(7) −9	(8) −100		

7—1. Integers—Multiplication (a)

Let us study the following multiplication facts :

$2 \times 4 = 8$

$2 \times 3 = 6$

$2 \times 2 = 4$

$2 \times 1 = 2$

$2 \times 0 = 0$

$2 \times - = ...$

We find that the first numbers remaining the same, the second numbers decrease by one. The next number in the series 4, 3, 2, 1, 0 will be -1. The products of the two numbers are 8, 6, 4, 2, 0. Each product is 2 less than the previous one. After 8, 6, 4, 2, 0, we will have -2.

We may therefore guess $2 \times (-1) = -2$.

Similarly, we may arrive at the following results.

$2 \times (-2) = -4$

$2 \times (-3) = -6$

The product of a positive integer and a negative integer is a negative integer.

Exercise 7—1

Find the product of :

(a) $4 \times (-3)$ (b) $5 \times (-4)$ (c) $7 \times (-5)$

(d) $8 \times (-6)$ (e) $9 \times (-7)$ (f) $10 \times (-8)$

(g) $11 \times (-9)$ (h) $2 \times (-3)$ (i) $3 \times (-6)$

(j) $5 \times (-10)$

7—2. Integers — Multiplication (b)

Look at the following pattern.

$4 \times 4 = 16$

$3 \times 4 = 12$

$2 \times 4 = 8$

$1 \times 4 = 4$

$0 \times 4 = 0$

The second number remains the same and the first numbers 4, 3, 2, 1, 0 are in descending order. We may find that -1 comes after 0. The products 16, 12, 8, 4, 0 are also in descending order and we may guess the next number as -4.

$$\therefore\ (-1) \times 4 = -4$$
$$\therefore\ (-2) \times 4 = -8$$
$$(-3) \times 4 = -12 \text{ and so on.}$$

In the previous lesson we have seen that $4 \times (-2) = -8$; $4 \times (-3) = -12$. We therefore find

$$4 \times (-2) = -8; \quad (-2) \times 4 = -8$$
$$4 \times (-3) = -12; \quad (-3) \times 4 = -12$$

Exercise 7—2

Find the product of :

(1) $(-10) \times 7$ (2) $(-9) \times 8$ (3) $(-8) \times 7$

(4) $(-7) \times 6$ (5) $(-6) \times 5$ (6) $(-5) \times 4$

(7) $(-4) \times 3$ (8) $(-3) \times 2$ (9) $(-11) \times 6$

(10) $(-12) \times 10$.

7—3. Integers — Multiplication (c)

We have already seen that

$$2 \times (-2) = -4$$
$$1 \times (-2) = -2$$
$$0 \times (-2) = 0$$

Following this pattern, we may guess that

$$(-1) \times (-2) = 2$$
$$(-2) \times (-2) = 4$$
$$(-3) \times (-2) = 6$$

The product of two negative integers is a positive integer.

Exercise 7—3

Find the product of :

(1) $(-10) \times (-11)$ (2) $(-12) \times (-10)$ (3) $(-9) \times (-8)$

(4) $(-8) \times (-7)$ (5) $(-6) \times (-10)$ (6) $(-12) \times (-4)$

(7) $(-7) \times (-9)$ (8) $(-6) \times (-5)$ (9) $(-11) \times (-8)$

(10) $(-11) \times (-10)$

7—4. Integers — Multiplication (d)

Let us now summarise what we have learnt about multiplication of two integers.

$$(+2) \times (+3) = 6$$

$$(+2) \times (-3) = -6$$

$$(-2) \times (+3) = -6$$

$$(-2) \times (-3) = 6$$

We learn the following :

1. The product of two positive integers is a positive integer.
2. The product of two negative integers is a positive integer.
3. The product of one positive integer and a negative integer is a negative integer.

These can be shown in a tabular column as follows.

×	Positive integer	Negative integer
Positive integer	Positive integer	Negative integer
Negative integer	Negative integer	Positive integer

Exercise 7--4

1. Find the product of :

(1) $2 \times (-3)$	(2) 3×2
(3) $(-4) \times (-3)$	(4) $(-3) \times (-5)$
(5) $(-8) \times 6$	(6) $7 \times (-9)$
(7) $(-5) \times (-10)$	(8) $(-4) \times 6$
(9) $(-7) \times 10$	(10) $(-11) \times (-8)$

2. Find the product of :

(1) $(-125) \times 8 \times (-4)$

(2) $(-16) \times (-625) \times (-2)$

(3) $7 \times (-2) \times (-8)$

(4) $(-7) \times 8 \times (-4)$

(5) $12 \times 8 \times (-25)$

(6) $6 \times 10 \times (-20)$

(7) $(-7) \times (-2) \times (-10)$

(8) $(-3) \times (-4) \times 8$

8. Integers — Division

We know that from every multiplication fact, two division facts can be obtained.

For example, from $6 \times 4 = 24$, we get,

$$6 = \frac{24}{4} \quad \text{and} \quad 4 = \frac{24}{6}$$

We have seen that $6 \times (-5) = -30$.

From this multiplication fact we can get

$(-30) \div 6 = -5$ and

$(-30) \div (-5) = +6$

Further, $(-7) \times (-4) = 28$. From this we can get,

$(+28) \div (-7) = -4$ and $+28 \div (-4) = -7$.

Similarly we may obtain the following results.

$(+12) \div (+3) = +4$
$(+12) \div (-3) = -4$
$(-12) \div (+3) = -4$
$(-12) \div (-3) = +4$

These can be put in the following tabular column :

Dividend Number	Divisor	Quotient
+ 12	+ 3	+ 4
+ 12	− 3	− 4
− 12	+ 3	− 4
− 12	− 3	+ 4

If the dividend and the divisor have like signs, the quotient is positive ; if the dividend and the divisor have unlike signs, the quotient is negative.

÷	Positive integer	Negative integer
Positive Integer	Positive integer	Negative integer
Negative integer	Negative integer	Positive integer

Exercise 8—1

1. Find the quotient :

(1) $5 \div 1$ (2) $5 \div (-1)$ (3) $(-5) \div (-1)$

(4) $(-5) \div 1$ (5) $0 \div 1$ (6) $0 \div (-1)$

2. Find the quotient :

(1) $20 \div 5$ (2) $16 \div 2$

(3) $30 \div (-6)$ (4) $60 \div (-12)$

(5) $(-80) \div (-8)$ (6) $(-133) \div (-7)$

(7) $(-196) \div (+7)$ (8) $(-208) \div 16$

(9) $0 \div 100$ (10) $0 \div (-100)$

(11) $(-80) \div 80$ (12) $80 \div (-80)$

3. **Find the quotient :**

(1) $\frac{12}{3}$ (2) $\frac{15}{-3}$

(3) $\frac{-15}{3}$ (4) $\frac{-15}{-3}$

(5) $\frac{-8}{2}$ (6) $\frac{-16}{-8}$

(7) $\frac{-20}{10}$ (8) $\frac{30}{-10}$

(9) $\frac{-50}{5}$ (10) $\frac{-100}{-10}$

Exercise 8—2

Find the pattern and fill up the following tabular columns

1.

+	−4	−3	−2	−1	0	1	2	3	4
4					4			7	8
3						4	5		
2					2				
1								4	5
0									
−1									
−2									
−3									
−4									

2.

×	−4	−3	−2	−1	0	1	2	3	4
4					0			12	
3						3			
2									8
1							2		
0						0		0	
−1									
−2									
−3									
−4									

Exercise 8—3

1. There are 100 questions that require the answers as either true or false in a Test paper. 2 marks will be awarded to each of the correct answers and −1 to each of the wrong answers and 0 to the questions not answered.

   (a) Sivakami wrote correct answers for 63 questions and wrong answers for 28 questions but failed to write answers for 9 questions. How many marks will she score?

   (b) Nalini wrote correct answers for 50 questions and wrong answers for the remaining questions. What is her score ?

2. A frog falls into a well 7 m deep. Every time when it tries to climb up it climbs up to a distance 70 cm but slips down 20 cm. Find the distance that the frog climbs up in one attempt and also how many times should it try to come out of the well.

3. Kari Nayanar who belonged to Korkkaiyur in the Chola kingdom composed the following on mensuration. This is in the book "Kanakkathikaram".

A callotes climbs up a palm tree of height 32 cubits. It goes up 12 finger spaces a day but slips down 4 finger spaces in every move. Find how many days will the callotes take to reach the top of the tree?

(24 finger spaces = 1 cubit)

TEST PAPER 2

1. Answer the following:
   (1) Which is the additive inverse of -5?
   (2) What is the additive inverse of 0?
   (3) Which is the greater of the following? -2, -3.
   (4) Which is smaller? 0, -5.
   (5) How many integers are there between -7 and 2?.

2. Match the following:

	A	B	
(1)	$(-13) + (-8)$	-15	( )
(2)	$(-13) - (-8)$	0	( )
(3)	$5 \times (-3)$	-1	( )
(4)	$15 \div (-15)$	-5	( )
(5)	$0 \times (-3)$	-21	( )
(6)	$(-5) \times (-3)$	1	( )
(7)	$\frac{-3}{-3}$	15	( )

3. **State whether true or false, giving reason :**

   (1) Inverse of 2 is -2.

   (2) Inverse of (-3) is (-3)

   (3) Inverse of $-(-5)$ is -5

   (4) Inverse of 0 is -0.

   (5) Inverse of $-(+2)$ is -2.

4. Fill in the blanks :

   (1) $(-8) - (-2) = (-8) + (\quad)$

   (2) $(-2) \times (-3) \times 6 = (\quad)$

   (3) $(-48) \div (-16) = (\quad)$

   (4) $(-8) + (-5) - (-13) = (\quad)$

   (5) $(-4)^8 = (\quad)$

BRAHMA GUPTA

Brahma Gupta, the great mathematician, who lived in the 6th century A. D. in our country is one of those who first discovered and introduced the integers to the world. He was the son of Jishnu. He was working as the chief astronomer at the space researh centre at Ujjain. He wrote his discoveries in his book called "Brahma Siddhantha", (sometimes called "Brahma Spuda Siddhantha") when he was aged thirty.

Brahma Siddhantha contains 21 chapters of which the first ten chapters deal with Astronomy. The 11th chapter contains his own rejections or modifications of the mathematical concepts in vogue. The 12th chapter contains his discoveries in Number Theory and Mensuration and the 21st chapter deals with spheres.

He also took a thorough study of integers and operations in integers, successors to integers, triangles, quadrilaterals and Number systems.

The integers are also called Brahma Gupta's numbers, as he was the first to make a detailed study of them.

Don't you want to become a great mathematician like Brahma Gupta? Therefore try!

ANSWERS

Exercise 4—1

1. (1) $<$ (2) $<$ (3) $>$ (4) $<$ (5) $>$ (6) $<$
2. (1) 300 (2) 20 (3) 299 (4) -14 (5) 0 (6) 0
3. (1) 0 (2) -5 (3) -2010 (4) -410
4. $-13, -8, -2, 0, 2, 3, 6, 10$
5. $11, 6, 2, 0, -1, -4, -6, -11$
6. (a) 8, (b) 4, (c) 3.

Exercise 5—4

2. (a) -6 (b) 4 (c) 7 (d) -3 (e) -8 (f) 10
   (g) -9 (h) 0

Exercise 5—5

2. (1) -6 (2) 6 (3) -28 (4) 35 (5) -100 (6) 0
   (7) 0 (8) -16 (9) -30 (10) -50

3. (a) $-3, -2, -1, 0, 1, 2, 3, 4$
   (b) $4, 3, 2, 1, 0, -1, -2, -3$
   (c) $(3, -3), (2, -2), (1, -1), (0, 0), (-1, 1), (-2, 2), (-3, 3)$
   (d) $(1, -3), (0, -2), (-1, -1), (-2, 0), (-3, 1)$

4. (i) $3 + 2 = 5$
   (ii) $3 + (-7) = -4$
   (iii) $3 + 4 = 7$
   (iv) $5 + (-3) = 2$
   (v) $(-4) + 8 = 4$
   (vi) $5 + 2 + (-6) = 1$

Exercise 5—6

2\. (1) 95 (2) −312 (3) −1027 (4) −300 (5) 100
(6) −100 (7) 300 (8) 600

Exercise 6—1

1\. (1), (2), (5), (6) — Addition
(3), (4), (7), (8) — Positive number

2\. (1), (2), (3) — Subtraction
(4), (5), (6) — Negative number

3\. (1) 2 (2) −4 (3) −5 (4) 7 (5) 0 (6) −8
(7) 9 (8) 100

Exercise 7—1

(a) −12 (b) −20 (c) −35 (d) −48 (e) −63
(f) −80 (g) −99 (h) −6 (i) −18 (j) −50

Exercise 7—2

(1) −70 (2) −72 (3) −56 (4) −42 (5) −30
(6) −20 (7) −12 (8) − 6 (9) −66 (10) −120

Exercise 7—3

(1) 110 (2) 120 (3) 72 (4) 56 (5) 60 (6) 48
(7) 63 (8) 30 (9) 88 (10) 110

Exercise 7—4

1\. (1) −6 (2) 6 (3) 12 (4) 15 (5) −48 (6) −63
(7) 50 (8) −24 (9) −70 (10) 88

2\. (1) 4000 (2) −20000 (3) 112 (4) 224
(5) −2400 (6) −1200 (7) −140 (8) 96

Exercise 8—1

1. (1) 5 (2) -5 (3) 5 (4) -5 (5) 0 (6) 0

2. (1) 4 (2) 8 (3) -5 (4) -5 (5) 10 (6) 19
   (7) -28 (8) -13 (9) 0 (10) 0 (11) -1 (12) -1

3. (1) 4 (2) -5 (3) -5 (4) 5 (5) -4 (6) 2
   (7) -2 (8) -3 (9) -10 (10) 10

Exercise 8—3

1. (a) 98 (b) 50
2. 50 cm, 16 times

TEST PAPER 2

1. (1) -5 (2) 0 (3) -2 (4) -5 (5) 8
2. (3), (5), (4), (2), (1), (7), (6)
3. (1) and (3) True
   (2), (4) and (5) False
4. (1) 2 (2) 36 (3) 3 (4) 0 (5) -64

3. RATIONAL NUMBERS

PRE TEST

Exercise 1

1. (a) Add : $\frac{1}{11} + \frac{1}{11} + \frac{1}{11} + \frac{1}{11} + \frac{1}{11}$

   (b) Multiply : $4 \times \frac{1}{7}$

   (c) Find the value of : $\frac{1}{11} \times 5$

   (d) Divide in fractional numbers : $4 \div 7$

2. Insert the suitable inequality sign :

   $\frac{4}{9}$ .... $\frac{4}{5}$

3. Rewrite in both ascending and descending orders :

   $\frac{3}{4}, \frac{2}{3}, \frac{1}{2}, \frac{1}{4}, \frac{5}{6}.$

4. Find the sum of : $2\frac{1}{4} + 3\frac{1}{2} + 5\frac{1}{8}$

5. Subtract : $6\frac{1}{12} - 3\frac{5}{6}$

6. Multiply : $2\frac{1}{3} \times 3\frac{1}{4}$

7. Find the product of : $5\frac{2}{7} \times 2\frac{2}{3} \times 3\frac{1}{2}$

8. Express the quotient of the following in fractional numbers:

   (1) $\frac{11}{21} \div \frac{22}{7}$

   (2) $5\frac{1}{4} \div 3\frac{1}{2}$

9. Simplify :

(1) $\left(2\frac{3}{4} \times 3\frac{7}{8}\right) + \left(2\frac{3}{4} \times 4\frac{1}{8}\right)$

(2) $\left(18 \times 5\frac{2}{9}\right) - \left(18 \times 4\frac{1}{9}\right)$

Summary of what you have learnt :

1. To find the sum of two like fractional numbers we add the numerators and put it as numerator and retain the denominator. Similarly to find the difference of two like fractions we find the difference of the two numerators and put it as numerator and retain the denominator.

2. To multiply a fractional number by a whole number, we multiply the numerator by that whole number.

3. To multiply a fractional number by another fractional number, we multiply the numerators and put the product as numerator and we multiply the denominators and put the product as denominator.

4. To divide a fractional number by another fractional number, we multiply the dividend fractional number by the reciprocal of the divisor fractional number.

PRE TEST

Exercise 2

1. State $\frac{1}{5}$ as a decimal fraction.
2. Tranform $\frac{3}{8}$ into a decimal fraction.
3. Convert 0·45 into a fractional number.
4. Express 0·00675 as a fractional number.
5. Express $\frac{3}{4}$ as a percentage.
6. State $\frac{1}{11}$ as a percentage.

7. Express $6\frac{2}{3}\%$ as a fractional number.

8. Change $14\frac{2}{7}\%$ into a fractional number.

9. Write 0·32 as percentage.

10. Express 2·025 as percentage.

11. Change 20% into a decimal fraction.

12. Change $12\frac{1}{2}\%$ into a decimal fraction.

13. State which is greater and express the greater in decimal fraction $\frac{15}{16}$, $\frac{7}{8}$

14. Find the sum of :

(a) 0·9237 + 0·256

(b) 49·206 + 2·0075

(c) 5·049 + 13·8765 + 9·37

15. Find the value of :

(a) 0·8792 − 0·498

(b) 39·0456 − 11·65497

16. Multiply :

(a) 3·9245 × 100

(b) 19·276 × 2·03

(c) 8·27 × 4·123 × 2·6

17. Compute :

(a) 32·056 ÷ 1000

(b) 208 ÷ 1·6

(c) 4·875 ÷ 0·125

18. Round 92567 (i) to the nearest tens (ii) to the nearest hundreds (iii) to the nearest thousands.

19. Express 13·08725 correct to (i) one decimal place (ii) two decimal places (iii) four decimal places.

20. Give the answer for $2{\cdot}35 \times 6{\cdot}28$ correct to two decimal places.

3—1. Negative rational numbers

You have seen in the chapter on integers that :

1. 300 m above mean sea level is denoted as +300 m and 152 m below mean sea level is denoted as −152 m.

2. Rs. 50 gain is noted as + Rs. 50 and Rs. 7 loss is noted as − Rs. 7.

3. 13 km to the east of a place is noted as + 13 km and 20 km to the west of a place is noted as − 20 km.

May be there is a place $100\frac{1}{2}$ m above the mean sea level as well as a place $225\frac{1}{4}$ m below the mean sea level.

As explained above, these can be represented as $+100\frac{1}{2}$ m and $-225\frac{1}{4}$ m respectively.

Similarly Rs. $10\frac{3}{4}$ gain and Rs. $9\frac{1}{2}$ loss can be denoted as + Rs. $10\frac{3}{4}$ and − Rs. $9\frac{1}{2}$ respectively.

A place which is $15\frac{7}{10}$ km towards east may be noted as $+15\frac{7}{10}$ km and $5\frac{3}{8}$ km towards west may be noted as $-5\frac{3}{8}$ km.

From the above examples we realise the need for negative rational numbers like $-225\frac{1}{4}$, $-9\frac{1}{2}$ and $-5\frac{3}{8}$.

The division of a whole number by a natural number is in the form of a fractional number.

Example :

$\frac{5}{6}$, $\frac{0}{7}$, $\frac{2}{1}$ are fractional numbers.

So also division of an integer by another non — zero integer is in the form of a rational number.

Example :

$$\frac{-2}{5}, \frac{3}{-7}, \frac{-5}{-6}$$

Note :

$-\frac{3}{7}, \frac{3}{-7}, \frac{-3}{7}$, All these represent the same rational number. Generally this is denoted as $-\frac{3}{7}$.

3—2. Positive rational numbers and negative rational numbers

You have learnt,

1. Positive integer × Positive integer = Positive integer

$$\therefore \frac{\text{Positive integer}}{\text{Positive integer}} = \text{Positive rational number}$$

Example :

$3 \times 5 = 15 \qquad \therefore\ 5 = \frac{15}{3}$

2. Positive integer × Negative integer
= Negative integer.

$$\therefore \frac{\text{Negative integer}}{\text{Positive integer}} = \text{Negative rational number}$$

$$\frac{\text{Negative integer}}{\text{Negative integer}} = \text{Positive rational number}$$

Example :

$$5 \times (-2) = (-10)$$

$$\therefore\ \frac{-10}{5} = -2; \quad \frac{-10}{-2} = 5$$

3. Negative integer × Negative integer
= Positive integer

$$\therefore \frac{\text{Positive integer}}{\text{Negative integer}} = \text{Negative rational number}$$

Example :

$$(-3) \times (-2) = +6$$

$$\therefore \quad \frac{6}{-3} = -2$$

Exercise 3—2

State whether the following are negative or positive rational numbers.

(1) $\frac{-5}{-2}$ (2) $\frac{-6}{-4}$ (3) $\frac{6}{-3}$ (4) $\frac{7}{2}$ (5) $-\frac{2}{3}$

(6) $\frac{-10}{15}$ (7) $\frac{-15}{-3}$ (8) $\frac{16}{-4}$ (9) $\frac{0}{-7}$ (10) $-\frac{10}{5}$

4. Rational Numbers—Inverse

$4 + 0 = 0 + 4 = 4$

Here 0 is the Additive identity.

$4 + (-4) = 0$ $\quad\therefore$ -4 is the inverse of $+4$

$(-5) + 5 = 0$ $\quad\therefore$ The inverse of -5 is $+5$

Similarly we can find the inverse of rational number also.

For example let us find the inverse of $\frac{5}{6}$

$$\frac{5}{6} + \left(\frac{-5}{6}\right) = 0$$

$\therefore$ The inverse of $\frac{5}{6}$ is $-\frac{5}{6}$

The inverse of $-\frac{2}{7}$ is $\frac{2}{7}$. How ?

Exercise 4—1

Find the inverse of :

(1) 0 (2) 1 (3) -100 (4) $\frac{5}{7}$

(5) $-\frac{9}{14}$ (6) $1\frac{5}{6}$ (7) $-2\frac{2}{3}$ (8) $\frac{9}{8}$

(9) $-\frac{11}{10}$ (10) $\frac{24}{17}$ (11) $\frac{-6}{7}$ (12) $\frac{7}{9}$

(13) $\frac{-8}{-4}$ (14) $-\frac{100}{91}$

5—1. Like Radional Numbers — Addition

Note the following number line :

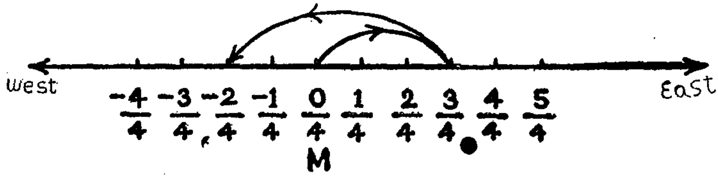


Fig. 3-1.

Pari starts from 0 and moves $\frac{3}{4}$ km towards east and then moves $\frac{5}{4}$ km towards west. You will see that he will be at $\frac{2}{4}$ km to the west of 0 $\left(\text{i.e. at } -\frac{2}{4}\right)$

Like wise,

$$\frac{5}{11} + \left(-\frac{2}{11}\right) = \frac{3}{11}$$

and $$\frac{6}{13} + \left(-\frac{8}{13}\right) = \frac{-2}{13}$$

Learn :

The sum of the rational numbers with the same denominators is,

(1) a rational number

(2) has the same denominator

(3) The numerator is the sum of the numerators.

Examples :

1. $\frac{11}{21} + \left(\frac{-15}{21}\right) = \frac{11 + (-15)}{21} = \frac{-4}{21} = -\frac{4}{21}$

2. $\left(\frac{-8}{13}\right) + \frac{4}{13} = \frac{(-8) + 4}{13} = \frac{-4}{13} = -\frac{4}{13}$

3. $\left(\frac{-2}{7}\right) + \left(\frac{-6}{7}\right) = \frac{(-2) + (-6)}{7} = \frac{-8}{7} = -\frac{8}{7}$

Exercise 5—1

Find the sum :

1. $\frac{3}{5} + \frac{4}{5}$
2. $\frac{4}{7} + \left(\frac{-5}{7}\right)$
3. $\left(\frac{-10}{13}\right) + \frac{4}{13}$
4. $\left(\frac{-4}{11}\right) + \left(\frac{-5}{11}\right)$
5. $\frac{15}{23} + \frac{11}{23}$
6. $\frac{10}{17} + \left(\frac{-4}{17}\right)$
7. $\left(\frac{-5}{9}\right) + \frac{2}{9}$
8. $\left(\frac{-3}{10}\right) + \left(\frac{-7}{10}\right)$
9. $\left(\frac{-2}{-11}\right) + \left(\frac{-3}{-11}\right)$
10. $\left(\frac{-7}{-20}\right) + \left(\frac{-6}{-20}\right)$

5—2. Unlike Rational Numbers — Addition

To add any two unlike rational numbers, we have to convert them into like rational numbers and then find their sum as done previously.

Example :

Find the sum of $\frac{5}{6} + \left(\frac{-7}{8}\right)$

The l. c. m of 6 and 8 is 24

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

$$\frac{-7}{8} = - \frac{-7 \times 3}{8 \times 3} = \frac{-21}{24}$$

Hence $\frac{5}{6} + \left(\frac{-7}{8}\right) = \frac{20 + (-21)}{24} = \frac{-1}{24} = -\frac{1}{24}$

Exercise 5—2

Find the sum :

1. $\frac{2}{3} + \frac{3}{4}$
2. $\frac{5}{6} + \frac{2}{3}$
3. $\frac{2}{7} + \left(-\frac{3}{5}\right)$
4. $\frac{2}{5} + \left(\frac{-3}{7}\right)$

5. $\left(\frac{-5}{12}\right) + \frac{1}{4}$

6. $\left(\frac{7}{-12}\right) + \frac{5}{8}$

7. $\left(\frac{7}{-8}\right) + \left(\frac{-5}{6}\right)$

8. $\left(-\frac{7}{10}\right) + \left(-\frac{4}{15}\right)$

9. $3\frac{1}{6} + 5\frac{5}{6}$

10. $11\frac{2}{7} + \left(-5\frac{3}{5}\right)$

11. $\left(-7\frac{2}{5}\right) + 6\frac{1}{4}$

12. $\left(-11\frac{7}{10}\right) + \left(-15\frac{4}{15}\right)$

5—3. Like Rational Numbers — Subtraction

Recall that when subtracting an integer, we added its **inverse.**

Examples :

(1) $\frac{5}{7} - \frac{4}{7}$

The inverse of $\frac{4}{7}$ is $\left(-\frac{4}{7}\right)$

$$\frac{5}{7} + \left(-\frac{4}{7}\right) = \frac{5 + (-4)}{7} = \frac{1}{7}$$

(2) $\frac{4}{11} - \left(\frac{-3}{11}\right)$

The inverse of $-\frac{3}{11}$ is $\frac{3}{11}$

$$\frac{4}{11} + \left(\frac{3}{11}\right) = \frac{4+3}{11} = \frac{7}{11}$$

(3) $\left(-\frac{6}{13}\right) - \frac{5}{13}$

The inverse of $\frac{5}{13}$ is $-\frac{5}{13}$

$$\therefore \left(-\frac{6}{13}\right) - \frac{5}{13} = \left(-\frac{6}{13}\right) + \left(-\frac{5}{13}\right)$$

$$= \frac{(-6) + (-5)}{13} = \frac{-11}{13} = -\frac{11}{13}$$

(4) $\left(-\frac{8}{17}\right)-\left(-\frac{9}{17}\right)$

The inverse of $-\frac{9}{17}$ is $+\frac{9}{17}$

$$\therefore \quad \frac{-8}{17}-\left(-\frac{9}{17}\right)=\frac{-8}{17}+\frac{9}{17}$$

$$=\frac{-8+9}{17}=\frac{1}{17}$$

Exercise 5—3

Fill the result :

1. $\frac{5}{11}-\frac{2}{11}$
2. $\frac{2}{3}-\frac{1}{3}$
3. $\frac{7}{17}-\left(\frac{-8}{17}\right)$
4. $\frac{8}{13}-\left(-\frac{9}{13}\right)$
5. $\left(\frac{-7}{12}\right)-\frac{4}{12}$
6. $\left(\frac{-13}{23}\right)-\frac{10}{23}$
7. $\left(-\frac{17}{37}\right)-\left(\frac{-21}{37}\right)$
8. $\left(\frac{-15}{31}\right)-\left(\frac{27}{-31}\right)$
9. $\left(\frac{-2}{-7}\right)-\left(\frac{-5}{-7}\right)$
10. $\left(\frac{-7}{-26}\right)-\left(\frac{5}{-26}\right)$

5—4. Unlike Rational Numbers - Subtraction

Example :

$$\left(-\frac{5}{12}\right)-\left(-\frac{7}{8}\right)=\ ?$$

The l. c. m. of 12 and 8 is 24.

$$\left(-\frac{5}{12}\right)-\left(-\frac{7}{8}\right)=-\frac{10}{24}-\left(-\frac{21}{24}\right)$$

The inverse of $-\frac{21}{24}$ is $\frac{21}{24}$

$$\therefore \quad \left(-\frac{5}{12}\right)-\left(-\frac{7}{8}\right)=\frac{-10+21}{24}$$

$$=\frac{11}{24}$$

Exercise 5–4

Find the result :

1. $\frac{3}{4} - \frac{1}{3}$
2. $\frac{5}{6} - \frac{3}{4}$
3. $\frac{5}{14} - \left(-\frac{5}{21}\right)$
4. $\frac{2}{3} - \left(\frac{-3}{5}\right)$
5. $\left(-\frac{3}{8}\right) - \frac{5}{6}$
6. $\left(-\frac{5}{6}\right) - \frac{3}{10}$
7. $\left(-\frac{7}{12}\right) - \left(-\frac{7}{30}\right)$
8. $\left(-\frac{8}{15}\right) - \left(-\frac{7}{30}\right)$
9. $8\frac{1}{6} - 5\frac{5}{6}$
10. $2\frac{2}{3} - 4\frac{1}{6}$
11. $\left(-3\frac{2}{3}\right) - \left(-5\frac{3}{10}\right)$
12. $\left(-7\frac{7}{12}\right) - 5\frac{3}{10}$
13. $15\frac{2}{15} - 11\frac{5}{12}$

5–5. Multiplication of a Rational Number by a Whole Number

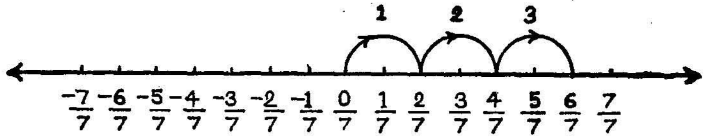


Fig. 3-2.

1. $1 \times \frac{2}{7} = \frac{2}{7}$
2. $2 \times \frac{2}{7} = \frac{4}{7}$
3. $3 \times \frac{2}{7} = \frac{6}{7}$

When you studied fractional numbers you have learnt that

$$3 \times \frac{2}{7} = \frac{3 \times 2}{7} = \frac{6}{7}$$

This is explained in Fig. 3-2.

Hence, to multiply a fractional number by a whole number we multiply its numerator by that whole number.

Example :

$$4 \times \frac{5}{11} = \frac{4 \times 5}{11} = \frac{20}{11}$$

Similarly a rational number can be multiplied by a whole number.

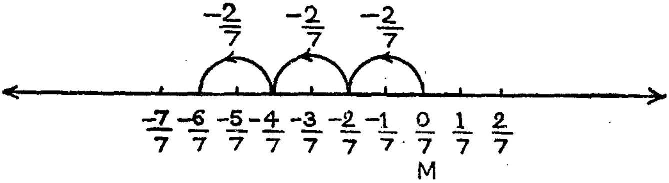


Fig. 3-3.

From the above figure it is seen that

$$3 \times \frac{-2}{7} = \frac{3 \times (-2)}{7} = \frac{-6}{7}$$

To multiply a negative rational number by a whole number it is enough to multiply its numerator by that whole number.

Example :

$$5 \times \left(\frac{-3}{11}\right) = \frac{5 \times (-3)}{11} = \frac{-15}{11}$$

Similarly to multiply a negative rational number by a negative integer multiply its numerator by that negative integer.

Example :

$$-3 \times \left(\frac{-4}{13}\right) = \frac{(-3) \times (-4)}{13} = \frac{12}{13}$$

Exercise 5–5

Find the products of :

1. $2 \times \frac{1}{7}$

2. $3 \times \left(\frac{-2}{5}\right)$

3. $(-4) \times \frac{2}{7}$

4. $(-3) \times \left(\frac{-5}{11}\right)$

5. $0 \times \frac{2}{7}$ 6. $0 \times \left(-\frac{3}{4}\right)$

7. $-1 \times \frac{2}{3}$ 8. $\frac{3}{4} \times (-1)$

9. $\left(-\frac{6}{11}\right) \times 4$ 10. $\left(-\frac{2}{7}\right) \times (-3)$

5—6. Multiplication of a Rational Number by another Rational Number

You know already that the product of two fractional numbers is

(1) a fractional number

(2) The numerator of the product is the product of the numerators

(3) The denominator of the product is the product of the denominators.

Example :

$$\frac{5}{4} \times \frac{3}{7} = \frac{5 \times 3}{4 \times 7} = \frac{15}{28}$$

Similarly the multiplication of rational numbers can be found.

Example :

(1) $\frac{5}{6} \times \left(-\frac{7}{9}\right) = \frac{5 \times (-7)}{6 \times 9} = \frac{-35}{54}$

(2) $\left(-\frac{2}{3}\right) \times \left(\frac{5}{7}\right) = \frac{(-2) \times 5}{3 \times 7} = \frac{-10}{21}$

(3) $\left(-\frac{3}{7}\right) \times \left(-\frac{5}{11}\right) = \frac{(-3) \times (-5)}{7 \times 11} = \frac{15}{77}$

Note : We keep the denominators always positive.

Exercise 5—6

1. Multiply and give the answers in the simplest form :

(1) $\frac{2}{9} \times \frac{2}{5}$ (2) $\frac{3}{7} \times \left(-\frac{2}{5}\right)$

(3) $\left(-\frac{3}{10}\right) \times \frac{7}{5}$ (4) $\left(-\frac{2}{11}\right) \times \frac{33}{52}$

(5) $\frac{5}{6} \times \frac{7}{11}$ (6) $\frac{6}{7} \times \left(-\frac{14}{18}\right)$

(7) $\left(-\frac{5}{13}\right) \times \frac{3}{10}$ (8) $\left(-\frac{6}{11}\right) \times \left(-\frac{2}{3}\right)$

2. Multiply and give the answers in the simplest form :

(1) $2\frac{1}{2} \times 1\frac{1}{7}$ (2) $3\frac{1}{3} \times 2\frac{2}{5}$

(3) $\left(1\frac{2}{3}\right) \times \left(-\frac{3}{10}\right)$ (4) $2\frac{1}{2} \times (-1)$

(5) $\left(-1\frac{1}{7}\right) \times 2\frac{4}{5}$ (6) $\left(-2\frac{2}{3}\right) \times 1\frac{1}{5}$

(7) $\left(-3\frac{2}{3}\right) \times \left(-2\frac{1}{7}\right)$ (8) $\left(-4\frac{1}{4}\right) \times \left(-3\frac{1}{2}\right)$

3. Multiply and give the answers in the simplest form :

(1) $4 \times \frac{7}{12} \times \frac{3}{5}$ (2) $3 \times \frac{1}{6} \times \frac{4}{5}$

(3) $6\frac{2}{3} \times \frac{7}{10} \times \left(-\frac{2}{3}\right)$

(4) $\left(-5\frac{2}{5}\right) \times \left(-2\frac{1}{6}\right) \times \frac{2}{3}$

(5) $\left(-1\frac{1}{3}\right) \times \left(-2\frac{1}{4}\right) \times \frac{7}{9}$

(6) $2\frac{1}{7} \times \left(-\frac{7}{15}\right) \times 2\frac{1}{4}$

(7) $4\frac{5}{6} \times \left(-\frac{6}{29}\right) \times 3\frac{1}{2}$

(8) $\frac{2}{7} \times \left(-1\frac{1}{2}\right) \times \left(-1\frac{2}{3}\right)$

(9) $\left(\frac{3}{11}\right) \times \left(-2\frac{1}{5}\right) \times \left(-\frac{5}{3}\right)$

(10) $\left(-\frac{5}{7}\right) \times \left(-1\frac{2}{5}\right) \times \left(-\frac{7}{12}\right)$

5—7. Rational Numbers—Division

Recall what yau have learnt about the division of fractional numbers previously.

Examples :

(1) Simplify : $\frac{2}{3} \div \frac{3}{4}$

We should multiply $\frac{2}{3}$ by the reciprocal of $\frac{3}{4}$ i. e. by $\frac{4}{3}$.

$$\therefore \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$$

(2) Simpilfy : $1\frac{5}{7} \div 2\frac{2}{3}$

$$1\frac{5}{7} = \frac{12}{7}; \quad 2\frac{2}{3} = \frac{8}{3}$$

$$\therefore 1\frac{5}{7} \div 2\frac{2}{3} = \frac{12}{7} \div \frac{8}{3}$$

Reciprocal of $\frac{8}{3}$ is $\frac{3}{8}$

$$\therefore 1\frac{5}{7} \div 2\frac{2}{3} = \frac{12}{7} \times \frac{3}{8} = \frac{36}{56}$$

The g. c. d. of 36 and 56 is 4.

$$\therefore \frac{36}{56} = \frac{4 \times 9}{4 \times 14} = \frac{9}{14}$$

The above method will also be applicable to the division of rational numbers.

(3) Simplify : $\frac{5}{7} \div \left(-\frac{2}{3}\right)$

The reciprocal of $-\frac{2}{3}$ is $-\frac{3}{2}$

$$\therefore \quad \frac{5}{7} \div \left(-\frac{2}{3}\right) = \frac{5}{7} \times \left(-\frac{3}{2}\right)$$

$$= \frac{5 \times (-3)}{7 \times 2} = \frac{-15}{14}$$

(4) Simplify : $\left(-4\frac{2}{3}\right) \div 1\frac{3}{4}$

$$-4\frac{2}{3} = \frac{-14}{3}; \quad 1\frac{3}{4} = \frac{7}{4}$$

The reciprocal of $\frac{7}{4}$ is $\frac{4}{7}$

$$\therefore \quad \left(-4\frac{2}{3}\right) \div 1\frac{3}{4} = \left(-\frac{14}{3}\right) \times \frac{4}{7}$$

$$= \frac{(-14) \times 4}{3 \times 7} = \frac{-56}{21}$$

The g. c. d. of 56 and 21 is 7

$$\therefore \quad \frac{-56}{21} = \frac{7 \times (-8)}{7 \times 3} = -\frac{8}{3}$$

Exercise 5—7

1. Find the reciprocals of :

(1) $\frac{2}{7}$ (2) $-\frac{3}{5}$ (3) $6\frac{1}{3}$ (4) $-11\frac{5}{9}$

(5) 1 (6) -1

2. Simplify :

(1) $2 \div \frac{3}{5}$ (2) $1 \div \frac{2}{3}$

(3) $0 \div \frac{2}{7}$ (4) $(-2) \div \frac{1}{2}$

(5) $3 \div \left(-\frac{3}{4}\right)$ (6) $(-5) \div \left(-\frac{2}{5}\right)$

(7) $(-4) \div \frac{2}{3}$ (8) $5 \div \left(-\frac{2}{5}\right)$

(9) $(-7) \div \left(-\frac{3}{4}\right)$ (10) $\left(-\frac{2}{11}\right) \div (-2)$

3. Simplify :

(1) $\frac{5}{7} \div \frac{4}{3}$ (2) $\frac{2}{11} \div \left(-\frac{3}{13}\right)$

(3) $\left(-\frac{2}{15}\right) \div \frac{3}{11}$ (4) $\left(-\frac{2}{7}\right) \div \left(-\frac{1}{3}\right)$

(5) $\frac{2}{7} \div \frac{5}{13}$ (6) $\frac{7}{13} \div \left(-\frac{3}{4}\right)$

(7) $\left(-\frac{6}{11}\right) \div \frac{5}{6}$ (8) $\left(-\frac{3}{10}\right) \div \left(-\frac{7}{9}\right)$

4. Simplify :

(1) $2\frac{1}{3} \div 3\frac{1}{2}$ (2) $\left(-3\frac{1}{7}\right) \div 2\frac{3}{4}$

(3) $3\frac{2}{3} \div \left(-1\frac{1}{4}\right)$ (4) $\left(-6\frac{1}{3}\right) \div \left(-2\frac{3}{4}\right)$

(5) $5\frac{1}{6} \div 2\frac{3}{4}$ (6) $\left(-2\frac{3}{10}\right) \div 3\frac{2}{5}$

(7) $11\frac{1}{2} \div \left(-3\frac{3}{4}\right)$ (8) $\left(-7\frac{2}{10}\right) \div \left(-4\frac{3}{5}\right)$

6. Order in Rational Nnumbers

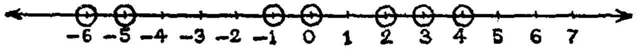


Fig. 3-4.

While we learnt about integers, we found that − 2 is greater than − 5, since − 2 is to the right of − 5 along the number line.

Hence $-2 > -5$

Since -2 is to the left side of 0, -2 is less than 0.

$\therefore \quad -2 < 0$

Hence the rational numbers -2, -5, 0 can be arranged as -5, -2, 0 in ascending order or 0, -2, -5 in descending order.

$$-5 < -2 < 0 \;; \qquad 0 > -2 > -5.$$

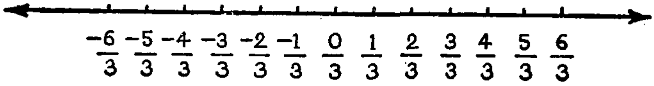


Fig. 3-5.

Similarly we may arrange rational numbers in order.

Example :

The rational number $-\frac{2}{3}$ is to the right of $-\frac{5}{3}$.

$$\therefore \quad -\frac{2}{3} > -\frac{5}{3}$$

The rational number $-\frac{4}{3}$ is to the left of 0.

$$\therefore \quad -\frac{4}{3} < 0$$

Let us consider the rational numbers having different denominators.

Example 1 :

Which is greater $-\frac{5}{7}$ or $-\frac{3}{5}$?

The l. c. m. of 7 and 5 is 35.

$$-\frac{5}{7} = \frac{-25}{35} \;; \quad -\frac{3}{5} = \frac{-21}{35}$$

$$-21 > -25$$

$$\therefore \quad \frac{-21}{35} > \frac{-25}{35}$$

That is, $-\frac{3}{5} > -\frac{5}{7}$

$\therefore \quad -\frac{3}{5}$ is greater.

Example 2 :

Which is smaller $-\frac{5}{8}$ or $-\frac{5}{12}$?

The l. c. m. of 8 and 12 is 24.

$$-\frac{5}{8} = \frac{-15}{24}; \qquad -\frac{5}{12} = \frac{-10}{24}$$

$$-15 < -10$$

$$\therefore \quad \frac{-15}{24} < \frac{-10}{24}$$

$\therefore \quad -\frac{15}{24}$ is smaller. That is, $-\frac{5}{8}$ is smaller.

Example 3 :

Which is greater $\frac{5}{-6}$ or $\frac{-3}{4}$?

$$\frac{5}{-6} = -\frac{5}{6}; \qquad \frac{-3}{4} = -\frac{3}{4}$$

The l. c. m. of 6 and 4 is 12.

$$\therefore \quad -\frac{5}{6} = -\frac{10}{12}; \qquad -\frac{3}{4} = -\frac{9}{12}$$

$$-9 > -10$$

$$\therefore \quad -\frac{9}{12} > -\frac{10}{12}$$

That is, $\frac{-3}{4}$ is greater.

Example 4 :

Which is the smallest?

$$\frac{-2}{3}; \; -\frac{5}{6}; \; \frac{7}{-8}$$

The l. c. m. of 3, 6 and 8 is 24.

$$\frac{-2}{3} = -\frac{2}{3} = -\frac{16}{24}$$

$$-\frac{5}{6} = -\frac{20}{24}$$

$$\frac{7}{-8} = -\frac{7}{8} = -\frac{21}{24}$$

$$-21 < -20 < -16$$

$\therefore$ -21 is the smallest.

Hence, $-\frac{21}{24}$ is the smallest.

$\therefore$ $\frac{7}{-8}$ is the smallest.

Note :

There are innumerable rational numbers between any two rational numbers. Hence we can't find the next rational number to $-\frac{1}{4}$

Example 5 :

Write in descending order.

$$-\frac{2}{5}, \; \frac{-3}{10}, \; \frac{4}{-15}, \; \frac{-3}{-5}$$

$$-\frac{2}{5}; \; \frac{-3}{10} = -\frac{3}{10}; \quad \frac{4}{-15} = -\frac{4}{15}; \; \frac{-3}{-5} = \frac{3}{5}$$

The l. c. m. of 5, 10 and 15 is 30

$$-\frac{2}{5} = -\frac{12}{30}; \; -\frac{3}{10} = -\frac{9}{30};$$

$$-\frac{4}{15} = -\frac{8}{30}; \; \frac{3}{5} = \frac{18}{30}$$

$$18 > -8 > -9 > -12$$

$$\therefore \ \frac{18}{30} > -\frac{8}{30} > -\frac{9}{30} > -\frac{12}{30}$$

$$\text{That is } \frac{-3}{-5} > \frac{4}{-15} > \frac{-3}{10} > -\frac{2}{5}$$

Exercise 6—1

1. Which is greater in each of the pairs?

(1) $\frac{5}{7}, \frac{4}{7}$ (2) $\frac{4}{11}, \frac{3}{11}$

(3) $\frac{-5}{8}, \frac{3}{4}$ (4) $-\frac{5}{8}, -\frac{3}{4}$

(5) $\frac{-15}{16}, \frac{7}{-8}$ (6) $\frac{-2}{3}, \frac{3}{5}$

(7) $-\frac{7}{10}, \frac{4}{-5}$ (8) $\frac{-7}{12}, -\frac{5}{8}$

2. Which is smaller in each of the following pairs?

(1) $1\frac{1}{3}, 1\frac{1}{5}$ (2) $2\frac{2}{3}, 2\frac{1}{4}$

(3) $-5\frac{1}{3}, -6\frac{1}{2}$ (4) $-\frac{5}{11}, -\frac{7}{8}$

(5) $\frac{-11}{12}, \frac{5}{-6}$ (6) $\frac{-7}{15}, \frac{5}{-8}$

3. Write in ascending order:

(1) $-\frac{2}{3}, \frac{-5}{6}, \frac{-2}{-2}, \frac{11}{-12}$

(2) $\frac{0}{-4}, \frac{-1}{-4}, \frac{-1}{2}, \frac{5}{-6}$.

4. Write in descending order:

(1) $\frac{0}{15}, \frac{-9}{10}, \frac{-14}{-15}, -\frac{4}{5}$

(2) $\frac{-2}{15}, -\frac{5}{12}, \frac{7}{-10}, \frac{-2}{-3}, \frac{3}{-4}$.

7. Decimal Fractions

7—1 (a). Changing ordinary fractions into decimal fractions

You have already learnt how to change ordinary fractional nnmeral into decimal fractional numeral. Let us recall it.

Example 1:

(1) $\frac{1}{2} = \frac{5}{10} = 0{\cdot}5$ (2) $\frac{1}{4} = \frac{25}{100} = 0{\cdot}25$

(3) $\frac{3}{4} = \frac{75}{100} = 0{\cdot}75$ (4) $\frac{1}{8} = \frac{125}{1000} = 0{\cdot}125$

(5) $\frac{1}{20} = \frac{5}{100} = 0{\cdot}05$ (6) $\frac{1}{125} = \frac{8}{1000} = 0{\cdot}008$.

First transform the given fractional numeral into equivalent fraction with the denominator as a power of 10. Then change it as decimal fraction. This method can easily be used in the case of fractional numerals having 2 and 5 as prime factors of the denominator.

Example 2:

Change $\frac{7}{40}$ as a decimal fraction.

$\therefore \frac{7}{40} = 0{\cdot}175$

```
      0·175
   ________
40 | 7·0
   | 4 0
   |______
   | 3 0 0
   | 2 8 0
   |______
   |   2 00
   |   2 00
   |______
```

Exercise 7—1 (a)

Write down as decimal fractions:

(1) $\frac{5}{8}$ (2) $\frac{7}{16}$ (3) $\frac{9}{250}$

(4) $\frac{11}{125}$ (5) $\frac{16}{625}$ (6) $\frac{7}{40}$

(7) $\frac{13}{80}$ (8) $\frac{17}{160}$ (9) $\frac{81}{320}$

(10) $\frac{127}{640}$

7—1 (b). Changing decimal fractions into ordinary fractions

You know the method of changing decimal fractions into ordinary fractions.

Example 1 :

Change 0·1 into ordinary fractional numeral.

$$0{\cdot}1 = \frac{1}{10}$$

Example 2 :

Change 0·07 into ordinary fractional numeral.

$$0{\cdot}07 = \frac{7}{100}$$

Example 3 :

Change 0·256 into ordinary fractional numeral.

$$0{\cdot}256 = \frac{256}{1000}$$

[The g. c. d. of the numerator and the denominator is 8. Hence divide by 8 and simplify.]

$$= \frac{32}{125}$$

Exercise 7—1 (b)

Write the following as ordinary fractional numeral :

(1) 0·7 (2) 0·13 (3) 0·127 (4) 0·003

(5) 0·2 (6) 0·002 (7) 0·25 (8) 0·32

(9) 0·128 (10) 0·375

7—1 (c). Changing percentages into decimal fractions

Example 1 :

Write 37% as a decimal fraction.

Percentage is a fraction with 100 as denominator.

$$\therefore\ 37\% = \frac{37}{100} = 0{\cdot}37$$

Example 2 :

Change 25% into a decimal fraction.

$$25\% = \frac{25}{100} = 0{\cdot}25$$

Exercise 7—1 (c)

Write as decimal fractions :

(1) 27% (2) 42% (3) 77% (4) 84% (5) 98%
(6) 99% (7) 18% (8) 8% (9) 2% (10) 1%.

7—1 (d). Comparing ordinary fractions by transforming them into decimal fractions

Example 1 :

Which is greater $\frac{1}{4}$ or $\frac{1}{2}$?

$$\frac{1}{4} = 0{\cdot}25 ; \quad \frac{1}{2} = 0{\cdot}5$$

$0{\cdot}5$ i.e., $\frac{1}{2}$ is greater.

Example 2 :

Write in the ascending order : $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{4}$.

$$\frac{1}{5} = 0{\cdot}2 ; \quad \frac{1}{8} = 0{\cdot}125 ; \quad \frac{1}{4} = 0{\cdot}25.$$

The least is $0{\cdot}125$ and the greatest is $0{\cdot}25$.

$\therefore \quad 0{\cdot}125 < 0{\cdot}2 < 0{\cdot}25$

i.e., $\frac{1}{8} < \frac{1}{5} < \frac{1}{4}$

Example 3 :

Write in the descending order: $\frac{5}{16}$, $\frac{3}{5}$, $\frac{7}{20}$.

$$\frac{5}{16} = 0{\cdot}3125$$

$$\frac{3}{5} = 0{\cdot}6$$

$$\frac{7}{20} = 0{\cdot}35$$

The greatest is 0·6 ; the least is 0·3125.

$\therefore \quad 0{\cdot}6 > 0{\cdot}35 > 0{\cdot}3125.$

i.e., $\frac{3}{5} > \frac{7}{20} > \frac{5}{16}$

Exercise 7—1 (d)

1. Which of the fractions of the following pairs are greater than the other? (Find by converting them into decimal fractions.)

(1) $\frac{7}{10}, \frac{9}{10}$ (2) $\frac{61}{100}, \frac{59}{100}$

(3) $\frac{127}{100}, \frac{128}{100}$ (4) $\frac{1}{2}, \frac{2}{5}$

(5) $\frac{7}{10}, \frac{3}{5}$ (6) $\frac{3}{8}, \frac{1}{4}$

(7) $\frac{11}{16}, \frac{5}{8}$ (8) $\frac{3}{16}, \frac{1}{4}$

(9) $\frac{1}{2}, \frac{3}{5}$ (10) $\frac{7}{16}, \frac{3}{8}$

2. Transform into decimal fractions and write in the ascending order :

(1) $\frac{7}{10}, \frac{3}{10}, \frac{9}{10}$ (2) $\frac{97}{100}, \frac{91}{100}, \frac{93}{100}$

(3) $\frac{991}{1000}, \frac{789}{1000}, \frac{987}{1000}$ (4) $\frac{3}{16}, \frac{1}{4}, \frac{3}{8}$

(5) $\frac{1}{2}, \frac{2}{5}, \frac{8}{25}$

3. Transform into decimal fractions and write in the descending order :

(1) $\frac{49}{100}, \frac{63}{100}, \frac{59}{100}$ (2) $\frac{11}{20}, \frac{7}{10}, \frac{9}{40}$

(3) $\frac{7}{10}, \frac{3}{8}, \frac{3}{4}$ (4) $\frac{17}{25}, \frac{13}{20}, \frac{97}{125}$

(5) $\frac{79}{32}, \frac{15}{16}, \frac{7}{8}$

7—2. Decimal fractions — Addition

You have already learnt the addition of decimal fractions. If a decimal fraction is added to another decimal fraction, the result is a decimal fraction.

Example 1 :

Add : 4·0927 + 5·9276

$$\begin{array}{r} 4{\cdot}0927\ + \\ 5{\cdot}9276 \\ \hline 10{\cdot}0203 \\ \hline \end{array}$$

Example 2 :

Add : 16·9201 + 127·08 + 9·456

$$\begin{array}{r} 16{\cdot}9201 \\ 127{\cdot}08 \\ 9{\cdot}456 \\ \hline 153{\cdot}4561 \\ \hline \end{array}$$

Exercise 7—2

Find the sum :

(1) 0 724 + 0·992

(2) 9·43 + 11·57 + 127·98

(3) 112·902 + 92·87 + 8·0276

(4) 7·0023 + 17·03 + 16 032 + 10·07602

(5) 3.8·09 + 426·81 + 0·276 + 0 0235

7—3. Decimal fractions — Subtraction

In the previous standard you have learnt the subtraction of decimal fractions. Recall it.

Example 1 :

From 15·03 subtract 9·5

$$\begin{array}{r} 15{\cdot}03\ - \\ 9{\cdot}5\ \ \\ \hline 5{\cdot}53\ \ \ \\ \hline \end{array}$$

Example 2 :

Find the answer : 112·523 – 79·9876

$$\begin{array}{r} 112{\cdot}5230\ - \\ 79{\cdot}9876\ \ \ \\ \hline 32{\cdot}5354\ \ \ \\ \hline \end{array}$$

Exercise 7—3

Find the answer :

(1) 9·89 – 7·45 (2) 19·52 – 18 98

(3) 27·91 – 19·873 (4) 67·478 – 59·98

(5) 79·2 – 14·7294

7—4. Decimal fractions — Multiplication

When two decimal fractions are multiplied, the product is a decimal fraction. Let us work out multiplication of decimal fractions.

Example 1 :

Find the product of 1·1 × ·12

$$1{\cdot}1 \times {\cdot}12 = \frac{11}{10} \times \frac{12}{100} = \frac{132}{1000} = 0{\cdot}132$$

Example 2 :

Commute : 274·63 × 0·1253

The total number of decimal places is 6

```
  27463
   1253
-------
  82389
 137315
 54926
27463
-------
34411139
```

∴ Product is 34·411139

Exercise 7—4

Find the product of :

(1) 27 × 3·4 (2) 0·02335 × 0·12

(3) 27·0051 × 3·007 (4) 91378·21 × 0·0031

(5) 6·020301 × 2·0202.

7—5. Decimal Fractions — Division

You have already learnt the division of one decimal fraction by another decimal fraction. Look at the following examples :

Example 1 :

$$\frac{5\cdot33}{1\cdot3} = \frac{5\cdot33 \times 10}{1\cdot3 \times 10}$$

$$= \frac{53\cdot3}{13} = 4\cdot1$$

Example 2 :

$$\frac{5\cdot61741}{0\cdot0123} = \frac{5\cdot61741 \times 10000}{0\cdot0123 \times 10000}$$

$$= \frac{56174\cdot1}{123} = 456\cdot7$$

Example 3 :

$$0{\cdot}0004125 \div 0{\cdot}00125$$

$$\frac{0{\cdot}0004125}{0{\cdot}00125} = \frac{0{\cdot}0004125 \times 100000}{0{\cdot}00125 \times 100000}$$

$$= \frac{41\ 25}{125} = 0{\cdot}33$$

Exercise 7—5

Divide :

(1) $0{\cdot}02376 \div 1000$ (2) $54{\cdot}72 \div 1{\cdot}2$

(3) $9{\cdot}936 \div 0{\cdot}016$ (4) $85{\cdot}1835 \div 0{\cdot}015$

(5) $0{\cdot}0004984 \div 0{\cdot}00014$

7—6. Approximation

Eaxmples :

(1) State to the nearest 10 :

475 ; 485

$475 \simeq 480$

$485 \simeq 480$

(2) Write to the nearest 100:

4853 ; 4850

$4853 \simeq 4900$

$4850 \simeq 4800$

(3) Give to the nearest 1000 :

5500 ; 4510

$5500 \simeq 6000$

$4510 \simeq 5000$

(4) Give to the nearest first decimal place :

0·75 ; 0·85 ; 0·96 ; 0·86

0·75 ≏ 0·8

0·85 ≏ 0·8

0·96 ≏ 1·0

0·86 ≏ 0·9

(5) Write to the nearest second decimal place :

0·7255 ; 0·725

0.7255 ≏ 0·73

0·725 ≏ 0·72

(6) Write correct to the third decimal place:

9·6509 ; 171·0543

9·6509 ≏ 9.651

171·0543 ≏ 171·054

Note :

From the above examples we derive the following rulcs :

1. If the digit in the place next to the number of places to be correctd is less than 5, then write the number upto that digit.

Examples

(1) 67·542. Write correct to the second decimal place.
67·542 ≏ 67·54 (Leave out 2 in the third decimal place)

(2) 0·7983. Write correct to the third decimal place.
0·7983 ≏ 0·798 (Leave out 3 in the fourth decimal place)

2. If the digit in the place next to the number of places to be corrected is more than 5 or is 5 followed by a digit other than 0, then add 1 to that digit and write the number upto that place.

Examples :

(1) 4·576. Give the answer correet to the second decimal place.

4·576.≏4 58 (The digit in the third decimal place is 6 > 5.)

(2) 8·451. Write correct to the first decimal place.

8·451 ≏ 8·5 (The number in the second and third decimal places is 51 > 50.)

3. If the digit in the place next to the number of places to be corrected is 5 or is 5 followed by 0, and if the preceding digit is (i) odd, add 1 to that digit and write the number upto that place (ii) even, write the number upto that digit without any change.

Examples :

(1) 4·75. Correct this to the first decimal place.

4·75 ≏ 4·8 (The number in the second decimal place is 5. The number in the first decimal place is 7, which is odd. Hence change it to 8.)

(2) 14·645. Correct this to the second decimal place.

14·645 ≏ 14·64 (The number in the third decimal place is 5. The number in the second decimal place is 4, which is even. Hence no change is needed.)

(3) 127·28502. Correct this to the second decimal place.

127·28502 ≏ 127·28 (The number in the third decimal place is 5 followed by 0. The number in the second decimal place is 8, which is even. Hence no change is needed.)

Exercise 7—6

1. Write the following to the nearest 10 :

(1)	45	(2)	55	(3)	94
(4)	87	(5)	97	(6)	1981

2. Write the following to the nearest 100 :

(1)	755	(2)	765	(3)	850
(4)	1948	(5)	1985	(6)	7350

3. Write the following to the nearest 1000 :

(1)	2345	(2)	2500	(3)	3500
(4)	4572	(5)	9500	(6)	9510

4. Write the following to the nearest first decimal place :

(1)	0·45	(2)	0·75	(3)	0·65
(4)	7·55	(5)	17·79	(6)	198·94

5. Write the following to the nearest second decimal place :

(1)	0·[illegible]55	(2)	0·645	(3)	0·7951
(4)	3·455	(5)	8·725	(6)	0·205

6. Give the answer correct to the third decimal place :

(1)	7·9635	(2)	9·8475	(3)	19·8705
(4)	1980·0006	(5)	0·0025	(6)	11·0734

7. State the answer correct to the second decimal place :

5·1234 + 7 0025 + 16·4

8. Find the answer correct to the third decimal place :

17·9298 − 8·9[illegible]26

9. Find the product correct to the fourth decimal place :

1·75 × 0·213

10. (i) $\frac{265}{1000}$; Write the answer correct to the second decimal place.

(ii) $\frac{6335}{10000}$; Give the answer correct to the third decimal place.

(iii) $126 \cdot 27 \div 153$. Give the result correct to the second decimal place.

7—7. Terminating and Non-terminating decimal fractions

You know to transform ordinary fractions into decimal fractions. Study the following examples.

(1) $\frac{1}{2} = 0 \cdot 5$ $\qquad \frac{1}{10} = 0 \cdot 1$

$\frac{1}{4} = 0 \cdot 25$ $\qquad \frac{1}{100} = 0 \cdot 01$

$\frac{1}{5} = 0 \cdot 2$

(2) $\frac{1}{3} = 0 \cdot 333 \ldots$ $\qquad \frac{1}{7} = 0 \cdot 142857142857 \ldots$

$\frac{1}{6} = 0 \cdot 166 \ldots$

When we transform ordinary fractions into decimal fractions, we get (1) terminating decimal fractions from the first set and (2) non-terminating repeating decimal fractions from the second set.

We have also non-terminating and non-repeating decimals. You will know about them in the higher standards.

Study the table below.

Dicimal fractions
- Terminating
- Non-terminating
  - Repeating
  - Non-repeating

If rational numbers are transformed into decimals then also we will get the same pattern.

TEST PAPER 3

[Time : 1 hour

1. Which of the following are true? Which are false?

   (1) 13 is a rational number.

   (2) $\frac{3}{2}$ is a natural numbes.

   (3) Every integer is a rational number.

   (4) Every rational number is an integer.

   (5) 0 is a rational number.

   (6) 0 is an integer.

   (7) The sum of two rational numbers is always a rational number.

   (8) The product of two rational numbers is always a rational number.

   (9) The opposite number of $\frac{-2}{7}$ is $\frac{-2}{7}$.

   (10) The multiplicative reciprocal of $\frac{-5}{3}$ is $\frac{-3}{5}$.

2. Reduce to the simplest form :

   (1) $\frac{-4914}{2340}$ (2) $\frac{-14580}{22932}$

3. Which is greater $\frac{-10}{11}$ or $\frac{7}{-9}$?

4. Which is smaller $\frac{12}{-13}$ or $\frac{-11}{12}$?

5. How many rational numbers are between $\frac{-2}{3}$ and $\frac{-1}{3}$?

6. Write in the descending order :

   $\frac{-2}{2}$, $\frac{-7}{12}$, $\frac{13}{4}$, $-2\frac{1}{2}$

7. Find the sum of

$$\frac{-7}{10}, \frac{5}{-12}, \frac{-5}{6}$$

8. Find the value of

$$\frac{-11}{12} - \frac{-7}{9}$$

9. Find the product of

$$\frac{-2}{7} \times \frac{-4}{9} \times \frac{5}{-3} \times \frac{-3}{-4}$$

10. Simplify :

$$\frac{92}{-25} \div \frac{-46}{150}$$

11. Transform $\frac{-5}{7}$ into a decimal fraction.

12. Change $\frac{-114}{375}$ into a decimal fraction.

13. Which of the following are terminating decimal fractions ? Which are non-terminating repeating decimal fractions ?

$$\frac{-5}{26}, \frac{-71}{-320}, \frac{-11}{80}, \frac{56}{-140}$$

ANSWERS

Exercise 1

1. (a) $\frac{5}{11}$ (b) $\frac{4}{7}$ (c) $\frac{5}{11}$ (d) $\frac{4}{7}$

2. $\frac{4}{9} < \frac{4}{5}$, $\frac{4}{5} > \frac{4}{9}$

3. Ascending order : $\frac{1}{4} < \frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{5}{6}$

Descending order : $\frac{5}{6} > \frac{3}{4} > \frac{2}{3} > \frac{1}{2} > \frac{1}{4}$

4. $10\frac{7}{8}$ 5. $2\frac{1}{4}$ 6. $7\frac{7}{12}$

7. $49\frac{1}{3}$ 8. (1) $\frac{1}{6}$ (2) $1\frac{1}{2}$

9. (1) 22 (2) 20

Exercise 2

1. ·2 2. ·375 3. $\frac{9}{20}$ 4. $\frac{27}{4000}$

5. 75% 6. $9\frac{1}{11}\%$ 7. $\frac{1}{15}$ 8. $\frac{1}{7}$

9. 32% 10. 202.5% 11. ·2 12. 0·125

13. $\frac{15}{16} > \frac{7}{8}$

14. (a) 1·1797 (b) 51·2135 (c) 28·2955

15. (a) 0·3812 (b) 27·39063

16. (a) 392·45 (b) 39·13028 (c) 88·652746

17. (a) 0·032056 (b) 130 (c) 39

18. (i) 92570 (ii) 92600 (iii) 93000

19. (i) 13·1 (ii) 13·09 (iii) 13·0872

20. 77·56

Exercise 3—2

(1), (3), (5), (6), (8), (10) Negative rational numbers

(2), (4), (7) Positive rational numbers

(9) Neither negative nor positive rational number

Exercise 5—1

1. $\frac{7}{5}$ 2. $\frac{-1}{7}$ 3. $\frac{-6}{13}$

4. $\frac{-9}{11}$ 5. $\frac{26}{23}$ 6. $\frac{6}{17}$

7. $\frac{-3}{9} = \frac{-1}{3}$ 8. $\frac{-10}{10} = -1$ 9. $\frac{-5}{-11} = \frac{5}{11}$

10. $\frac{-13}{-20} = \frac{13}{20}$

Exercise 5—2

(1) $1\frac{5}{12}$ (2) $1\frac{1}{2}$ (3) $\frac{-11}{35}$

(4) $-\frac{1}{35}$ (5) $-\frac{1}{6}$ (6) $\frac{1}{24}$

(7) $-1\frac{17}{24}$ (8) $\frac{-29}{30}$ (9) 9

(10) $5\frac{24}{35}$ (11) $-1\frac{3}{20}$ (12) $-26\frac{29}{30}$

Exercise 5—3

(1) $\frac{3}{11}$ (2) $\frac{1}{3}$ (3) $\frac{15}{17}$ (4) $1\frac{4}{13}$

(5) $\frac{-11}{12}$ (6) -1 (7) $\frac{4}{37}$ (8) $\frac{12}{31}$

(9) $-\frac{3}{7}$ (10) $\frac{6}{13}$

Exercise 5—4

(1) $\frac{5}{12}$ (2) $\frac{1}{12}$ (3) $\frac{25}{42}$ (4) $1\frac{4}{15}$

(5) $-1\frac{5}{24}$ (6) $-1\frac{2}{15}$ (7) $-\frac{7}{20}$ (8) $-\frac{3}{10}$

(9) $2\frac{1}{3}$ (10) $-1\frac{1}{2}$ (11) $1\frac{19}{30}$ (12) $-12\frac{53}{60}$

(13) $3\frac{43}{60}$

Exercise 5—5

(1) $\frac{2}{7}$ (2) $-\frac{6}{5}$ (3) $-\frac{8}{7}$ (4) $\frac{15}{11}$

(5) 0 (6) 0 (7) $-\frac{2}{3}$ (8) $-\frac{3}{4}$

(9) $-\frac{24}{11}$ (10) $\frac{6}{7}$

Exercise 5—6

1. (1) $\frac{4}{45}$ (2) $\frac{-6}{35}$ (3) $-\frac{21}{50}$ (4) $-\frac{3}{26}$

   (5) $\frac{35}{66}$ (6) $-\frac{2}{3}$ (7) $-\frac{3}{26}$ (8) $\frac{4}{11}$

2. (1) $2\frac{6}{7}$ (2) 8 (3) $-\frac{1}{2}$ (4) $-2\frac{1}{2}$

   (5) $-3\frac{1}{5}$ (6) $-3\frac{1}{5}$ (7) $7\frac{6}{7}$ (8) $14\frac{7}{8}$

3. (1) $1\frac{2}{5}$ (2) $\frac{2}{5}$ (3) $-3\frac{1}{9}$ (4) $7\frac{4}{5}$

   (5) $2\frac{1}{3}$ (6) $-2\frac{1}{4}$ (7) $-3\frac{1}{2}$ (8) $\frac{5}{7}$

   (9) 1 (10) $-\frac{7}{12}$

Exercise 5—7

1. (1) $\frac{7}{2}$ (2) $-\frac{5}{3}$ (3) $\frac{3}{19}$ (4) $-\frac{9}{104}$

   (5) 1 (6) -1

2. (1) $3\frac{1}{3}$ (2) $1\frac{1}{2}$ (3) 0 (4) -4

   (5) -4 (6) $12\frac{1}{2}$ (7) -6

   (8) $-12\frac{1}{2}$ (9) $9\frac{1}{3}$ (10) $\frac{1}{11}$

3. (1) $\frac{15}{28}$ (2) $-\frac{26}{33}$ (3) $-\frac{22}{45}$

   (4) $\frac{6}{7}$ (5) $\frac{26}{35}$ (6) $-\frac{28}{39}$

   (7) $-\frac{36}{55}$ (8) $\frac{27}{70}$

4. (1) $\frac{2}{3}$ (2) $-1\frac{1}{7}$ (3) $-2\frac{14}{15}$ (4) $2\frac{10}{33}$

(5) $1\frac{29}{33}$ (6) $\frac{-23}{34}$ (7) $-3\frac{1}{15}$ (8) $1\frac{13}{23}$

Exercise 6—1

1. (1) $\frac{5}{7}$ (2) $\frac{4}{11}$ (3) $\frac{3}{4}$ (4) $\frac{-5}{8}$

(5) $\frac{7}{-8}$ (6) $\frac{3}{5}$ (7) $-\frac{7}{10}$ (8) $\frac{-7}{12}$

2. (1) $1\frac{1}{5}$ (2) $2\frac{1}{4}$ (3) $-6\frac{1}{2}$ (4) $-\frac{7}{8}$

(5) $\frac{-11}{12}$ (6) $\frac{5}{-8}$

3. (1) $\frac{11}{-12}$, $\frac{-5}{6}$, $-\frac{2}{3}$, $\frac{-2}{-2}$

(2) $\frac{5}{-6}$, $\frac{-1}{2}$, $\frac{0}{-4}$, $\frac{-1}{-4}$

4. (1) $\frac{-14}{-15}$, $\frac{0}{15}$, $-\frac{4}{5}$, $\frac{-9}{10}$

(2) $\frac{-2}{-3}$, $\frac{-2}{15}$, $-\frac{5}{12}$, $\frac{7}{-10}$, $\frac{3}{-4}$

Exercise 7—1 (a)

(1) 0·625 (2) 0·4375 (3) 0·036

(4) 0·088 (5) 0·0256 (6) 0·175

(7) 0·1625 (8) 0·10625 (9) 0·253125

(10) 0·1984375

Exercise 7—1 (b)

(1) $\frac{7}{10}$ (2) $\frac{13}{100}$ (3) $\frac{127}{1000}$ (4) $\frac{3}{1000}$

(5) $\frac{1}{5}$ (6) $\frac{1}{500}$ (7) $\frac{1}{4}$ (8) $\frac{8}{25}$

(9) $\frac{16}{125}$ (10) $\frac{3}{8}$

Exercise 7—1 (c)

(1) 0·27 (2) 0·42 (3) 0·77 (4) 0·84

(5) 0·98 (6) 0·99 (7) 0·18 (8) 0·08

(9) 0·02 (10) 0·01

Exercise 7—1 (d)

1. (1) $\frac{9}{10}$ (2) $\frac{61}{100}$ (3) $\frac{127}{1000}$

(4) $\frac{1}{2}$ (5) $\frac{7}{10}$ (6) $\frac{3}{8}$

(7) $\frac{11}{16}$ (8) $\frac{1}{4}$ (9) $\frac{3}{5}$ (10) $\frac{7}{16}$

2. (1) $\frac{3}{10}, \frac{7}{10}, \frac{9}{10}$ (2) $\frac{91}{100}, \frac{93}{100}, \frac{97}{100}$

(3) $\frac{789}{1000}, \frac{987}{1000}, \frac{991}{1000}$ (4) $\frac{3}{16}, \frac{1}{4}, \frac{3}{8}$

(5) $\frac{8}{25}, \frac{2}{5}, \frac{1}{2}$

3. (1) $\frac{63}{100}, \frac{59}{100}, \frac{49}{100}$ (2) $\frac{7}{10}, \frac{11}{20}, \frac{9}{40}$

(3) $\frac{3}{4}, \frac{7}{16}, \frac{3}{8}$ (4) $\frac{97}{125}, \frac{17}{25}, \frac{13}{20}$

(5) $\frac{15}{16}, \frac{29}{32}, \frac{7}{8}$

Exercise 7—2

(1) 1·716 (2) 148·98 (3) 213·7996

(4) 50·14032 (5) 745·1995

Exercise 7—3

(1) 2·44 (2) 0·54 (3) 8·037

(4) 7·498 (5) 64·4706

Exercise 7—4

(1) 91·8 (2) 0·002814 (3) 81·2043357

(4) 283·272451 (5) 12·1622120802

Exercise 7—5

(1) 0·00002376 (2) 45·6 (3) 621

(4) 5678·9 (5) 3·56

Exercise 7—6

1.	(1) 40	(2) 60	(3) 90
	(4) 90	(5) 100	(6) 1980
2.	(1) 800	(2) 800	(3) 800
	(4) 1900	(5) 2000	(6) 7400
3.	(1) 2000	(2) 2000	(3) 4000
	(4) 5000	(5) 10000	(6) 10000
4.	(1) 0·4	(2) 0·8	(3) 0·6
	(4) 7·6	(5) 17·8	(6) 198·9
5.	(1) 0·66	(2) 0·64	(3) 0·80
	(4) 3·46	(5) 8·72	(6) 0·20
6.	(1) 7·964	(2) 9·848	(3) 19·870
	(4) 1980·001	(5) 0·002	(6) 11·073
7.	28·53	8. 8·937	9. 0·3728
10.	(1) 0·26	(2) 0·634	(3) 0·83

TEST PAPER 3

1. (1), (3), (5), (6), (7), (8), (10) **True**
   (2), (4), (9) **False**

2. (1) $-\frac{21}{10}$ (2) $-\frac{405}{637}$ 3. $\frac{7}{-9}$

4. $\frac{12}{-13}$ 5. Innumerable rational numbers

6. $\frac{13}{4}$, $\frac{-7}{12}$, $\frac{-2}{2}$, $-2\frac{1}{2}$ 7. $-\frac{117}{60}$ 8. $-\frac{5}{36}$

9. $-\frac{10}{63}$ 10. 12 11. $-0{\cdot}714285$ 12. $--0{\cdot}304$

13. Terminating decimal fractions:

$$\frac{-71}{-320},\ \frac{-11}{80},\ \frac{56}{-140}$$

Non-terminating repeating decimal fraction:

$$\frac{-5}{26}$$

4. NUMBER THEORY

1—1. Place holders in Equations (a)

Let us consider the addition of 4 and 5. That is, find the sum of 4 and 5. This problem can be rewritten as $4 + 5 = \square$, where we have to put the proper number in the place of □. The symbol '□' is therefore called a place holder. In the above problem this holds the place of 9. The same can be written as follows:

(1) $4 + 5 = \triangle$

(2) 4 + 5 = ⬡

(3) $4 + 5 = \bigcirc$

(4) 4 + 5 = 🔒

(5) $4 + 5 = ?$

All the geometrical symbols and the question mark are place holders and they hold the place of 9 in the above equations. Fig. 4-1 depicts another way of looking at this.

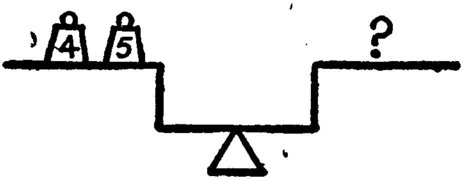


Fig. 4-1.

Exercise 1—1

1. Make equations using any of the place holders given (□, △, ⬡, ⊙ 🔒) for the following situations :

(1) Write the sum of 4 and 6.

(2) What should we get when 7 is added to 3 ?

(3) Find the sum of 5 and 11.

(4) Write the sum of 10 and 15.

(5) What should we get when 19 is added to 20 ?

2. Put the appropriate number in the place holder :

(1) $2 + 8 = \square$

(2) $87 + 13 = \triangle$

(3) $188 + 312 = \bigcirc$

(4) $220 + 780 = $ ▯

(5) $325 + 675 = $ ⬡

1—2. Place holders in Equations (b)

Can you say what should be added to 5 to get 12 ? This can be written in the following forms.

(1) $5 + \square = 12$

(2) $5 + \triangle = 12$

(3) $5 + $ ⬡ $ = 12$

(4) $5 + $ ▯ $ = 12$

(5) $5 + ? = 12$

Exercise 1—2

1. Write the following using any place holder :

(1) What number should be added to 12 to get 15 ?

(2) What natural number should we add to 10 to get 24 ?

(3) What whole number should be added to 9 to get 20 ?

(4) I have Rs. 6. How much should I save to increase it to Rs. 25 ?

(5) There are 39 pupils in a class. How many pupils are to be admitted to make the strength 45 ?

2. **Fill up :**

(1) 12 + □ = 16
(2) 17 + △ = 20
(3) 9 + ⬡ = 10
(4) 8 + ○ = 20
(5) 13 + ▯ = 25

1—3. Place holders in Equations (c)

The product of 6 and 3 can be written as follows:

(1) 6 × 3 = △
(2) 6 × 3 = ⬡
(3) 6 × 3 = □
(4) 6 × 3 = ▯
(5) 6 × 3 = ?

Exercise 1—3

1. Write the following statements using a place holder:

(1) Find the product of 5 × 6

(2) Find the product of 7 × 10

(3) I have 10 marbles. Narayanan has 3 times of what I have. How many marbles does Narayanan have?

(4) What is the cost of 6 pens at the rate of Rs. 5 per pen ?

(5) My friend weighs 3 times my weight. If my weight is 30 kg what is the weight of my friend ?

1—4. Place holders in Equations (d)

Example 1 :

Which two numbers can give a sum of 20 ?

This can be written as follows:

(1) △ + □ = 20
(2) ⬡ + △ = 20
(3) ▽ + ○ = 20
(4) ⬡ + ▯ = 20
(5) ? + ★ = 20

Example 2 :

Which two numbers have the difference of 10 ?

The above problem can be written using the place holders as follows :

(1) □ − △ = 10

(2) □ − ▽ = 10

(3) ⬡ − □ = 10

(4) ⊙ − ▢ = 10

(5) ? − ★ = 10

Exercise 1−4

1. Write using place holders only :

(1) Which two numbers give a sum of 10 ?

(2) The sum of 2 natural numbers is 100. Which are the two numbers ?

(3) I bought 50 fruits of which some are apples and some mangoes.

(4) Both John and Jone have a sum of Rs. 1000.

(5) Abdulla and Ali rear 548 hens totally.

2. Write as mathematical statement :

(1) □ + △ = 10

(2) ⬡ + △ = 20

(3) △ + ⬡ = 39

(4) ⊙ + ▢ = 103

(5) ? + ★ = 1,000

1—5. Place holders in Equations (e)

Example :

"The product of any two numbers" may be written as follows using place holders :

(1) □ × △ = 0
(2) △ × ⬡ = 10
(3) △ × ⬡ = 100
(4) ? × ○ = 1,000
(5) ★ × 🔒 = 10,000

Exercise 1—5

1. Write the following using place holders only :
   (1) What is 10 times 5 ?
   (2) What should be added to 8 to get 20 ?
   (3) What is the sum of 13 and 20 ?
   (4) I have 12 pencils. My friend has 12 times of what I have. How many pencils does my friend have ?
   (5) Which two numbers give the product 30 ? Which two numbers should be added to get 20 ?

2. Write some mathematical statements expressed by the following equations :

   (1) □ + △ = 100
   (2) ⊙ + 5 = 11
   (3) ⬡ + 4 = 32
   (4) ? + △ = 45
   (5) 8 + 22 = 🔒

1—6. Filling the Place holders with integers — Addition (a)

Example 1 :

4 + 6 = □
4 + 6 = 10

We get 10 when 6 is added to 4.

Example 2 :

8 + (−11) = △

8 + (−11) = −3

It is clear that we get (−3) when (−11) is added to 8.

Example 3 :

$(-10) + (-7) = \bigcirc$

$(-10) + (-7) = -17$

We get (– 17) when (– 7) is added to (– 10).

Exercise 1–6

Fill in the place holders using integers to make each of the following statements true:

(1) $6 + 8 = \square$

(2) $(-8) + 9 = \triangle$

(3) $(-11) + (-2) = \bigcirc$

(4) $4 + (-9) = ?$

(5) $(-13) + (-17) = \square$

(6) $(-12) + 3 = ?$

(7) $(-8) + (-2) = \bigcirc$

(8) $(-20) + (8) = \triangle$

(9) $(-8) + 8 = \triangleleft$

(10) $7 + 0 = \bigcirc$

1–7. Filling the Place holders with integers — Addition (b)

Example 1 :

$13 + \triangle = 20$

What shuuld be added to 13 to get 20 ?

Can you get 20 when 7 is added to 13 ?

$\therefore \quad 13 + 7 = 20$

Example 2 : Solve (using integers) :

$\square + 11 = 90$

What should be added to 11 to get 90 ?

79 should be added to 11 to get 90.

$\therefore \quad 79 + 11 = 90$

Fxercise 1—7

Answer using integers:

(1)	$4 + \triangle = 11$	(2)	$\square + 12 = 100$
(3)	$8 + \bigcirc = 4$	(4)	$\square + 11 = 8$
(5)	$12 + ? = 21$	(6)	$27 + \triangleleft = 0$
(7)	$30 + \square = 10$	(8)	$(+20) + \triangle = 20$
(9)	$5 + \bigcirc = 1$	(10)	$28 + \triangle = 100$

1—8. Filling the place holders — Multiplication

Example :

Answer using natural numbers :

$4 \times 2 = \triangle$

The product of 4 and 2 is 8

$\therefore \quad 4 \times 2 = 8$

The solution is 8.

Exercise 1—8

Solve using natural numbers :

(1) $6 \times 5 = \square$

(2) $7 \times 3 = \triangle$

(3) $9 \times 14 = \bigcirc$

(4) $25 \times 12 = \triangle$

(5) $125 \times 32 = \nabla$

(6) $625 \times 64 = ?$

1—9. Filling two place holders — Addition

Example :

Solve using natural numbers :

$\square + \triangle = 14$

That is, 'By adding which two natural numbers we get a sum of 14?'

Note the following table :

□	+	△	=	14					
13	+	1	=	14	6	+	8	=	14
12	+	2	=	14	5	+	9	=	14
11	+	3	=	14	4	+	10	=	14
10	+	4	=	14	3	+	11	=	14
9	+	5	=	14	2	+	12	=	14
8	+	6	=	14	1	+	13	=	14
7	+	7	=	14					

There are 13 solutions for the above addition and they are : (13, 1), (12, 2), (11, 3), (10, 4), (9, 5), (8, 6), (7, 7), (6, 8), (5, 9), (4, 10), (3, 11), (2, 12), (1, 13).

Any pair makes the equation true.

Exercise 1—9

Solve using natural numbers :

(1) □ + △ = 5

(2) △ + ⬡ = 10

(3) ⬡ + ⊙ = 7

(4) ? + ★ = 9

(5) ⬒ + △ = 12

1—10. Filling two place holders — Multiplication

Example :

Solve using natural numbers :

□ × △ = 15

This can be written as "Which two natural numbers give the product 15 ?"

□	×	△	=	15					
1	×	15	=	15	3	×	5	=	15
5	×	3	=	15	15	×	1	=	15

We get the following 4 soultions in natural numbers :

(1, 15), (3, 5) (5, 3), (15, 1)

Exercise 1—10

1. Find the solution in natural numbers :

(1) △ × □ = 16
(2) △ × □ = 20
(3) □ × ○ = 26
(4) ⊙ × ▭ = 24
(5) ? × ★ = 48

2. Solve using integers :

(1)	□	+	8	=	10	(2)	△	+	12	=	4
(3)	6	+	⊙	=	13	(4)	7	+	○	=	2
(5)	84	+	16	=	?	(6)	13	×	12	=	△
(7)	15	×	▽	=	120	(8)	△	×	11	=	132
(9)	△	+	?	=	14	(10)	▭	+	▽	=	22

1—11. Using letters instead of place holders

Example :

Instead of using different figures as place holders, a letter can also be used as a place holder.

For example,

$3 + x = 5$

Any alphabet y, a, b, ..., etc can also be used.

$3 + x = 5$ is an equation

Likewise 4 × □ = 12 can be written as

$4x = 12$

□ + △ = 9 can be written as

$x + y = 9$

$x + 3 = 5$ and $4x = 12$ are equations.

In the above x is called a variable.

$x + y = 9$ is also an equation. Here x and y are variables.

Exercise 1—11

Using $x, y, a, b,$ etc., as variables rewrite the sums contained in the Exercises 1—1 to 1—10.

2—1. *Solutions of equations* $[x + a = b]$

Example 1 :

Solve : $x + 3 = 5$

We have learnt that $x + 3 = 5$ is an equation. The equation can be solved by giving a suitable value to x, which makes the equation true.

When $x = 1$, $\quad 1 + 3 = 4; \quad 4 \neq 5$

$\therefore$ 1 is not a solution.

When $x = 2$, $\quad 2 + 3 = 5; \quad 5 = 5$

Hence 2 is a solution.

Let us now find out the method of solving equation.

$x + 3 = 5$. To remove 3 which is with x, add its additive inverse (-3) to both sides of the equation.

The equation becomes

$$x + 3 + (-3) = 5 + (-3)$$
$$x + 0 = 2$$
$$\therefore \; x = 2$$

The solution of

$$x + 3 = 5 \text{ is } 2$$

Verification :

Substitute 2 for x in the equation. The L. H. S. will be $2 + 3 = 5$. Both sides are equal. $\therefore$ The solution is 2.

Example 2 :

Find the solution in natural numbers :

$y - 6 = 11.$

By adding the inverse of -6 i.e., $(+6)$ to both the sides we get the following equation.

$$y - 6 + (+6) = 11 + (+6)$$
$$y + 0 = 17$$
$$\therefore \quad y = 17$$

Verification :

Substitute 17 for y in the equation. On L. H. S. we get $17 - 6 = 11$. 11 is there on both sides. Hence the solution is correct.

Exercise 2-1

Find the solution in natural numbers :

1. $x + 8 = 10$	2. $y + 3 = 4$
3. $z + 2 = 8$	4. $a + 5 = 11$
5. $b + 6 = 12$	6. $+ 2 + c = 10$
7. $m + 5 = 11$	8. $+ 8 + k = 14$
9. $14 + l = 15$	10. $+ 20 + p = 200$

2—2. Finding the solutions using integers [$x + a = b$]

Example :

$x + 5 = 2$

The inverse of $+5$ is -5. Add -5 to both sides. The solution is obtained.

$$x + 5 + (-5) = 2 + (-5)$$
$$x + 0 = -3$$
$$x = -3$$

$\therefore$ The solution is -3

Verification :

Put $x = -3$ in the given equation and the L. H. S. becomes $-3 + 5 = 2$.

2 is on the right side also.

$\therefore$ The equation is true for $x = -3$.

Exercise 2—2

Find the solution in integers:

1.	$x + 4 = 4$	2.	$y + 8 = +2$
3.	$z + 10 = 1$	4.	$m + 100 = +20$
5.	$+98 + x = 100$	6.	$k + 11 = +3$
7.	$a + 2 = +1$	8.	$+6 + b = +2$
9.	$25 + c = 15$	10.	$27 + l = +7$

2—3. Finding the solution in natural numbers $[ax = b]$

Recall : The multiplicative inverse of 2 is $\frac{1}{2}$

$\therefore \quad 2 \times \frac{1}{2} = 1 ; \quad \frac{1}{2} \times 2 = 1$

Example :

Solve : $2x = 6$

The multiplicative inverse of 2 is $\frac{1}{2}$

Multiplying both sides by $\frac{1}{2}$, the equation becomes

$$\frac{1}{2} \times 2x = \frac{1}{2} \times 6$$
$$1 \times x = 3$$
$$x = 3$$

$\therefore$ The solution is 3.

Verification :

$$L.H.S. = 2 \times 3 = 6$$
$$R.H.S. = 6$$

Hence the solution is correct.

Exercise 2—3

Find the solution in natural numbers :

1.	$4x$	=	16	2.	$5x$	=	75
3.	$7x$	=	343	4.	$13y$	=	208
5.	$14y$	=	154	6.	$12y$	=	864
7.	$11z$	=	132	8.	$10z$	=	10,000
9.	$100m$	=	100	10.	$25k$	=	125

2—4. Finding the solution in fractional numbers $[ax = b]$

Example :

Solve : $3x = 5$.

The multiplicative inverse of 3 is $\frac{1}{3}$.

Multiply both sides by $\frac{1}{3}$.

$$3x \times \frac{1}{3} = 5 \times \frac{1}{3}$$
$$1 \times x = \frac{5}{3}$$
$$\therefore x = \frac{5}{3}$$

The solution is $\frac{5}{3}$.

Verification :

L. H. S. $= 3 \times \frac{5}{3} = 5$

R. H. S. $= 5$

Hence the solution is correct.

Exercise 2—4

Find the solution in fractional numbers :

1. $2x = 7$
2. $3x = 6$
3. $4x = 7$
4. $5y = 2$
5. $6y = 13$
6. $7y = 21$
7. $8z = 32$
8. $9z = 99$
9. $8z = 17$
10. $11a = 9$

2—5. Finding the solution in Rational Numbers $[ax + b = c]$

Example :

Solve : $4x + 3 = 1$

Add the additive inverse of 3 on both sides. Now we get,

$$4x + 3 + (-3) = 1 + (-3)$$
$$4x + 0 = -2$$
$$4x = -2$$

The multiplicative inverse of 4 is $\frac{1}{4}$

$$\frac{1}{4} \times 4x = -2 \times \frac{1}{4}$$
$$x = -\frac{1}{2}$$

Verification :

$$\text{L. H. S.} = 4 \times \left(-\frac{1}{2}\right) + 3 = -2 + 3 = 1$$

$\text{R. H. S.} = 1$

Hence the solution is correct.

Exercise 2—5

Solve using Rational Numbers :

1. $2x + 5 = 9$
2. $4x + 8 = 24$
3. $7y + 1 = 24$
4. $8y + 9 = 8$
5. $11z + 13 = 10$
6. $9z + 10 = 8$
7. $a + 11 = 11$
8. $6p + 9 = 9$
9. $15c + 21 = 45$
10. $12b + 132 = 52$

2—6. Finding the solutions in integers $[x^2 = a]$

Example 1 :

We know $5^2 = 25$

$(-5)^2 = (-5)(-5) = 25$

In the equation $x^2 = 25$, 5, -5 both make the equation true

since $(+5)^2 = (+5)(+5) = 25$

and $(-5)^2 = (-5)(-5) = 25$

$\therefore$ The solutions of $x^2 = 25$ are $+5$ and -5.

Example 2 :

Let us solve $x^2 = 16$

Step I : Product of $(+4)$, $(+4)$ is 16

Step II : Product of (-4), (-4) is 16

Hence the solutions are $+4$ and -4.

Exercise 2—6

Solve : (Find the solutions in integers)

1. $x^2 = 4$	2. $a^2 = 36$
3. $l^2 = 49$	4. $b^2 = 225$
5. $k^2 = 484$	6. $m^2 = 625$
7. $c^2 = 169$	8. $d^2 = 144$
9. $t^2 = 64$	10. $y^2 = 121$

3. Equations — Applications

Example 1 :

Latha and Sudha have got together a sum of Rs. 50. Latha has got Rs. 20. How much does Sudha have ?

Latha's amount + Sudha's amount = Rs. 50

Let Sudha's amount be Rs. x

Latha's amount is Rs. 20

Hence $x + 20 = 50$

$$x + 20 + (-20) = 50 + (-20)$$

$$x + 0 = 30$$

$$x = 30$$

Sudha has got Rs. 30.

Verification :

$20 + 30 = 50$

Example 2 :

My age is 44. At present my age is 4 times my son's age. What is the age of my son ?

Let my son's age be x years.

$\therefore$ My age $= 4 \times$ (my son's age)

$= 4 \times x$ years

Hence $4 \times x = 44$

Multiply both sides by $\frac{1}{4}$

$$\frac{1}{4} \times 4x = \frac{1}{4} \times 44$$

$$x = 11$$

$\therefore$ My son's age is 11.

Verify the answer.

Example 3 :

The cost of an apple is 10 ps. more than what 7 bananas cost. If an apple costs Rs. 1·50, what is the cost of a banana ?

Let the cost of a banana be x paise.

According to the sum the cost of an apple = Rs. 1·50

$$= 150 \text{ ps.}$$

$$\text{That is, } 7x + 10 = 150$$

$$7x + 10 + (-10) = 150 + (-10)$$

$$7x = 140$$

$$7x \times \frac{1}{7} = 140 \times \frac{1}{7}$$

$$x = 20$$

The cost of one banana is 20 ps.

Exercise 3—1

1. The total amount that Chitra and Pankaja have got is Rs. 100. Pankaja has got Rs 60. Find what Chitra has got?

2. The total amount that Kalyani and Chellammal have got is Rs. 2,500. Chellammal has got Rs. 800. Find out what Kalyani has got?

3. The measure of an angle is 42. Find the measure of its complement.

4. The measure of an angle is 67. Find the measure of its supplement.

5. Find the side of a square whose perimeter is 160 m.

6. Find the side of a square whose perimeter is 350 m.

7. Find Priya's age if her father's age is 3 times her age. Her father's age is 51.

8. At present Raman's grand mother's age is 4 times that of Raman. If her age is 60, find Raman's age.

9. When 7 is added to 5 times a number, we get 17. What is that number?

10. When 100 is added to 11 times a number, we get 78. What is that number ?

11. Radha pays to the shop Rs. 1,000 and gets Rs. 40 back after purchasing 8 sarees of equal value. Find the price of one saree.

12. Nalini purchases 12 books of the same price. She gets Rs. 4 back when she gives Rs. 100. What is the cost of each book ?

13. Find the edge of a square field whose area is 900 m^2.

14. Find the edge of a square field whose area is 3025 m^2.

TEST PAPER 4 [Time 1 hour

I. (1) Rewrite using a place holder. What is the sum of 72 and 28 ?

(2) Fill up suitably with a natural number : $5 + \bigcirc = 13$.

(3) Fill up suitably with a natural number : $7 \times \triangle = 63$.

(4) Fill up suitably with natural numbers : $\triangle \times \square = 6$.

(5) Fill up suitably with natural numbers : $\triangle \times \square = 21$.

(6) Solve using rational numbers :

(1) $4x = 3$	(2) $2x + 3 = 5$
(3) $3x + 2 = 21$	(4) $5x + 1 = 21$
(5) $5x + 24 = 4$	(6) $x + 4 = 2$
(7) $13x = 26$	(8) $3x = 13$
(9) $2x = 3$	(10) $x + 14 = 4$

(7) The measure of an angle is 72°. What is the measure of its complement ?

(8) Find the edge of a square field whose area is 5625 sq. m.

(9) Find the number of marbles that Meera has got, given that, when 30 marbles are added to 7 times of what Meera has got, there will be 170 marbles.

(10) Umamaheswari purchased 15 books of equal price. She paid Rs. 50 to the shopkeeper and got back Rs. 20. What is the cost of each book ?

II. Match the following so as to make the equations true :

	(a)	(b)	
(1)	$x + 4 = 0$	1	( )
(2)	$x + 2 = 2$	5	( )
(3)	$x + 3 = 4$	-4	( )
(4)	$2x + 3 = -1$	-2	( )
(5)	$x + 4 = 20$	0	( )
(6)	$4x = 20$		

III. (1) Find out if there is any solution in whole number, for the following equation. State reasons.

$$2x + 7 = 2$$

(2) Find if there is any solution in integers for the equation $x + 4 = 4$. State reasons.

(3) Solve using natural numbers :

$$x^2 = 441$$

(4) Solve using rational numbers :

$$14x + 20 = 15$$

(5) It will amount to Rs. 700 if Rs. 200 is added to 5 times of what Maheswari has got. Find out how much Maheswari has got.

(6) Solve using rational numbers :

$$75 \times \Delta = 72$$

(7) *Using integers* solve,

$$x + 8 = 12$$

4—1. Inequations

So far we have studied about equations and their solutions. Let us now learn about inequations and their solutions.

Example :

$x = 2$ is an equation

$x < 2$, $x > 2$ are inequations

Examples :

(1) Try to tell a natural number greater than 2. The problem is to find x such that $x > 2$ and x is a natural number.

We find when x takes the values 3, 4, 5, 6, ... the inequation $x > 2$ is true.

(2) Find the solution in natural numbers for $x < 10$.

It is easily seen that 1, 2, ..., 9 are solutions of x.

(3) Find the solution in natural numbers for $x + 4 > 5$

When we substitute 2, 3, 4, ... for x we get numbers greater than 5 on the L. H. S.

For value 1, the inequation $x + 4 > 5$ is not true.

Hence the solutions are 2, 3, 4, ... There are innumerable solutions for this inequation.

Note : $x + 4 > 5$

$x + 4 + (-4) > 5 + (-4)$

$\therefore \; x > 1$

That is, the solution is a natural number greater than 1. They are 2, 3, 4, ...

Exercise 4—1

Solve using integers :

1. $x + 4 > 4$
2. $x + 8 > 12$
3. $x + 20 > 25$
4. $x + 100 > 900$
5. $x + 95 > 980$
6. $x + 201 > 1026$

4—2. Solving inequations — Multiplication

Example 1 :

Find the solution in natural numbers for

$$5x > 25$$

$$\frac{1}{5} \times 5x > \frac{1}{5} \times 25$$

$$\therefore \; x > 5$$

The solution is a natural number greater than 5 i. e. 6, 7, 8, 9, ...

Example 2 :

Find the solution in fractional numbers for

$$7x > 31$$

Multiply both sides by the multiplicative inverse of 7.

Thus $\frac{1}{7} \times 7x > \frac{1}{7} \times 31$

$$\therefore \; x > \frac{31}{7}$$

$\therefore$ The solution is fractional numbers greater than $4\frac{3}{7}$

Exercise 4—2

Find the solutions of the following, using fractional numbers :

1.	$4x > 12$	2.	$3x > 27$	3.	$7x > 61$
4.	$9x > 64$	5.	$11x > 432$	6.	$12x > 583$
7.	$13x > 651$	8.	$2x > 9$	9.	$5x > 20$
10	$6x > 25$				

4—3. Solving inequations using whole numbers — Addition

Example 1 :

Find the solution of $x + 5 < 6$ using whole numbers.

$$x + 5 + (-5) < 6 + (-5)$$

$$x < 1$$

∴ The solution is 0, since 0 is the only whole number less than 1.

Example 2 :

Solve : $x + 6 < 10$

$$x + 6 + (-6) < 10 + (-6)$$

$$x < 4$$

The solutions are 0, 1, 2 and 3.

Exercise 4—3

Find the solutions using whole numbers :

1.	$x + 3 < 4$	2.	$x + 6 < 8$
3.	$x + 9 < 12$	4.	$x + 21 < 25$
5.	$x + 42 < 47$	6.	$x + 59 < 65$
7.	$x + 67 < 74$	8.	$1923 + x < 1931$
9.	$2001 + x < 2010$	10.	$3895 + x < 3905$

5. Framing Formulae

I. First let us consider the perimeter of the squares of sides 2, 3, 5 units respectively.

Perimeter of the 1st square $= 2 + 2 + 2 + 2$
$= 4 \times 2$

Perimeter of the 2nd square $= 3 + 3 + 3 + 3$
$= 4 \times 3$

Perimeter of the 3rd square $= 5 + 5 + 5 + 5$
$= 4 \times 5$

$\therefore$ Perimeter of the square $= 4 \times$ the length of its side

If one of its side measures 'a' units then its perimeter $= 4a$

Then $P = 4a$ is the formula for finding the perimeter of a square.

P denotes the perimeter of the square and a, the side.

II. A car travels at a speed of 10 m per second. Find the distance travelled in 2 seconds, 3 seconds and 4 seconds by the car.

Distance travelled in 1 sec. $= 1 \times 10$ m.

Distance travelled in 2 sec. $= 2 \times 10$ m.

Distance travelled in 3 sec. $= 3 \times 10$ m.

Distance travelled in 4 sec. $= 4 \times 10$ m.

$\therefore$ Distance $=$ time travelled $\times$ speed

Denote the speed as u, time travelled as t and the distance covered in that time as s.

We get the formula $s = ut$, to find the distance covered in a specified time at a given speed.

III. Let us recall how we found the simple interest.

Principal (in Rs.)	Time (years)	Rate of interest	Simple interest (in Rs.)
1	1	·01	$1 \times 1 \times {\cdot}01$
1	2	·01	$1 \times 2 \times {\cdot}01$
1	2	·08	$1 \times 2 \times {\cdot}03$
5	2	·08	$5 \times 2 \times {\cdot}08$
500	3	·01	$500 \times 3 \times {\cdot}01$
500	4	·01	$500 \times 4 \times {\cdot}01$
1	1	i	$1 \times 1 \times i$
1	n	i	$1 \times n \times i$
P	n	i	Pni

If we denote the simple interest as I, then we get the formula $I = Pni$, where P denotes the principal, n the number of terms, i the rate of interest per term.

IV. Let us look at profit and loss problems.

Cost Price (in Rs.)	Selling Price (in Rs.)	Gain (in Rs.)	Gain Percent (%)
5	6	$6 - 5$	$\frac{6-5}{5} \times 100$
20	22	$22 - 20$	$\frac{22-20}{20} \times 100$
400	450	$450 - 400$	$\frac{450-400}{400} \times 100$
c	s	$s - c$	$\frac{s-c}{c} \times 100$

∴ The formula for finding the percentage of gain may be given by $g = \frac{s - c}{c} \times 100$

where s = selling price,

c = cost price,

and g = percentage of gain.

V. If there is loss in a sale, let us calculate the percentage of loss.

Cost price (in Rs.)	Selling price (in Rs.)	Loss (in Rs.)	Loss Percent (%)
10	8	10 − 8	$\frac{10 - 8}{10} \times 100$
100	95	100 − 95	$\frac{100 - 95}{100} \times 100$
300	200	300 − 200	$\frac{300 - 200}{300} \times 100$
c	s	$c - s$	$\frac{c - s}{c} \times 100$

$$l = \frac{c - s}{c} \times 100$$

where c = cost price,

s = selling price,

and l = percentage of loss.

Evaluating Formulae

Example :

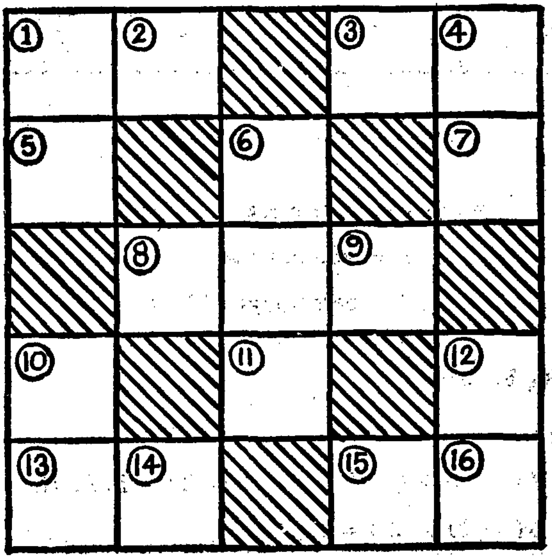
In $M = Vd$, $V = 20$, $d = 2{\cdot}5$. Find M.

$M = 20 \times 2{\cdot}5 = 50$

Exercise 5—1

1. Find M if $V = 100$, $d = \cdot 07$ in $M = Vd$
2. Find A if $b = 16$, $h = 6$ in $A = bh$
3. Find A if $b = 13$, $h = 7$ in $A = \frac{1}{2} bh$
4. Find A if $\pi = \frac{22}{7}$, $r = 7$ in $A = \pi r^2$
5. Find C if $\pi = 3{\cdot}14$, $r = 10$ in $C = 2\pi r$
6. Find V if $\pi = 3{\cdot}14$, $r = 10$, $h = 8$ in $V = \pi r^2 h$
7. Find V if $\pi = \frac{22}{7}$, $r = 7$, $h = 9$ in $V = \frac{1}{3} \pi r^2 h$
8. Find S if $a = 11$ in $S = 6a^2$
9. Find s if $u = 10$, $t = 6$ in $s = ut$
10. Find V if $\pi = 3{\cdot}14$; $r = 30$ in $V = \frac{4}{3} \pi r^3$

RIDDLE IN MATHEMATICS



Fill up the blanks with the help of the following clues :

ACROSS

(1) Solution of $x + 6 = 30$

(3) $4 - (-9)$

(5) $(-4) \times (-2)$

(6) $\frac{1}{4}$ th of -12

(7) Square root of 36 (in natural numbers)

(8) $3 \times 10^2 + 7 \times 10 + 8$

(10) Inverse of -1

(11) Solution of $a - 6 = 0$

(12) $\left(-\frac{5}{3}\right) \div \left(-\frac{5}{18}\right)$

(13) 8^2

(15) $100 + (-16)$

DOWN

(1) $3^3 + 1$

(2) $(-8) \div (-2)$

(3) Multiplicative Identity

(4) Area of a square whose side is 6 m = ..., sq. m.

(6) $3 \times 100 + 7 \times 10 + 6$

(8) Predecessor of 4

(9) Successor of 7

(10) Solution of $2x + 3 = 35$

(11) $\frac{3}{4} = \frac{\square}{8}$

(12) $\left(-6\frac{2}{5}\right) \times (-10)$

(14) $\left(\frac{-11}{-13}\right) + \left(\frac{-7}{13}\right) = \frac{\square}{13}$

(15) Solution of $3a = 24$

C. F. GAUSS

Gauss was born at Brunswick in Germany on 23rd April 1777. He was a very famous mathematician in his days. His father was an ordinary worker, who wanted Gauss to take to his own profession. But Gauss was very much interested in studies. It is told that he had known about numerals even before he started to speak. While his father was calculating the daily wages, the three year old boy pointed out the error in a moment. Some say that Gauss discovered the formula

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$

even when he was in the 1st standard.

At the age of 10, he was having clear ideas about Binomials and infinite series. Then he had close contact with Barteles, a school mathematics teacher in Brunswick and exchanged similar mathematical concepts. Barteles took him to Ferdinand, the Prince of Brunswick, introduced Gauss to the prince and arranged for financial aid to continue his higher education. He was first sent to the college in the year 1792 and became a member of Gottingen University. He conducted research on "Fundamental Theorems in Number Theory" and took his doctoral degree in 1799 at Helmstad.

He worked as a teacher first and then became the President and Professor of Mathematics of the Astronomical Research Centre of Gottingen University. There he led a pious and simple life and also published several findings in mathematics.

It may also be noted that the famous Mathematicians such as Dirichlet and Riemann studied under him and became popular only through his guidance.

He wore simple clothes. At the age of 60 he became a scholar in Russian Language. He also established a new and simple proof for his own "Fundamental Theorems in Number Theory".

No field in Mathematics was left untouched by Gauss. It was he who said "Mathematics is the queen of sciences and the Number Theory is the queen of Mathematics." E.T. Bell, therefore called him " Prince of Mathematics

During his period "Number system" was systamatized and considered as a separate subject of study. The contributions of Gauss to Mathematical knowledge finds an important place in the golden books of Mathematical History.

This famous mathematician passed away on 23rd February, 1855. The Germans by releasing the fascimile of that great mathematician added one more feather to their caps.

ANSWERS

Exercise 1—6

(1) 14 (2) 1 (3) − 13 (4) − 5 (5) − 30 (6) − 9 (7) − 10 (8) − 12 (9) 0 (10) 7

Exercise 1—7

(1) 7 (2) 88 (3) − 4 (4) − 3 (5) 9 (6) − 27 (7) − 20 (8) 0 (9) − 4 (10) 72

Exercise 1—8

(1) 30 (2) 21 (3) 126 (4) 300 (5) 4000 (6) 40000

Exercise 1—10

2. (1) 2 (2) −8 (3) 7 (4) −5 (5) 100 (6) 156 (7) 8 (8) 12 (9) (0, 14), (1, 13),... (14, 0), (15, −1), (−1, 15),... (16, −2), (−2, 16),... (30, −16), (−16, 30), ... (10) (−1, −22), (−22, −1), (− 2, −11), (−11, −2), (1, 22), (22, 1), (11, 2), (2, 11)

Exercise 2—1

(1) 2 (2) 1 (3) 6 (4) 6 (5) 6 (6) 8 (7) 6 (8) 6 (9) 1 (10) 180

Exercise 2—2

(1) 0 (2) − 6 (3) − 9 (4) − 80 (5) 2 (6) − 8 (7) − 1 (8) − 4 (9) − 10 (10) − 20

Exercise 2—3

(1) 4 (2) 15 (3) 49 (4) 16 (5) 11 (6) 72 (7) 12 (8) 1000 (9) 1 (10) 5

Exercise 2—4

(1) $3\frac{1}{2}$ (2) 2 (3) $1\frac{3}{4}$ (4) $\frac{2}{5}$ (5) $2\frac{1}{6}$ (6) 3 (7) 4 (8) 11 (9) $2\frac{1}{8}$ (10) $\frac{9}{11}$

Exercise 2—5

(1) 2 (2) 4 (3) $3\frac{2}{7}$ (4) $-\frac{1}{8}$ (5) $-\frac{3}{11}$ (6) $-\frac{2}{9}$

(7) 0 (8) 0 (9) $1\frac{3}{5}$ (10) $-6\frac{2}{3}$

Exercise 2—6

(1) 2, − 2 (2) 6, − 6 (3) 7, − 7 (4) 15, − 15
(5) 22, − 22 (6) 25, − 25 (7) 13, − 13 (8) 12, − 12
(9) 8, − 8 (10) 11, − 11

Exercise 3—1

(1) Rs. 40 (2) Rs. 1700 (3) 48° (4) 113° (5) 40 m

(6) $87\frac{1}{2}$ m (7) 17 (8) 15 (9) 2 (10) − 2 (11) Rs. 120

(12) Rs. 8 (13) 30 m (14) 55 m

TEST PAPER 4

I. (1) 72 + 28 = □ (2) 8 (3) 9 (4) (1, 6), (2, 3), (3, 2), (6, 1) (5) (1, 20), (2, 19), ... (20, 1) (6) (1) $\frac{3}{4}$ (2) 1

(3) $6\frac{1}{3}$ (4) 4 (5) −4 (6) −2 (7) 2 (8) $4\frac{1}{3}$ (9) $1\frac{1}{2}$

(10) − 10

(7) 18° (8) 75 m. (9) 20 (10) Rs. 8

II. (3), (6), (1), (4), (2)

III. (1) No. The solution $-\frac{5}{2}$ is a rational number.

(2) Yes. The solution 0 is an integer.

(3) 21 (4) $\frac{-5}{14}$ (5) Rs. 100 (6) $\frac{24}{25}$ (7) 4

Exercise 4—1

(1) 1, 2, 3, ... (2) 5, 6, 7, ... (3) 6, 7, 8, ...
(4) 801, 802, 803, ... (5) 886, 887, 888, ...
(6) 826, 827, 828, ...

Exercise 4—2

(1) $x > 3$ (2) $x > 9$ (3) $x > 8\frac{5}{7}$

(4) $x > 7\frac{1}{9}$ (5) $x > 39\frac{3}{11}$ (6) $x > 48\frac{7}{12}$

(7) $x > 50\frac{1}{13}$ (8) $x > 4\frac{1}{2}$ (9) $x > 4$

(10) $x > 4\frac{1}{6}$

Exercise 4—3

(1) 0 (2) 1, 0 (3) 2, 1, 0 (4) 3, 2, 1, 0

(5) 4, 3, 2, 1, 0 (6) 5, 4, 3, 2, 1, 0 (7) 6, 5, ... 1, 0

(8) 7, 6, 5, ... 1, 0 (9) 8, 7, ... 1, 0 (10) 9, 8, ... 1, 0

Exercise 5—1

(1) 7 (2) 96 (3) $45\frac{1}{2}$ (4) 154

(5) 62·8 (6) 2512 (7) 462 (8) 726

(9) 60 (10) 113040

5. GRAPHS

1—1. One Dimensional Graphs — Equations

We made use of number lines to study about integers. Let us now solve equations such as $x + 4 = 6$, $x + 6 = 4$, with the help of number lines.

Example 1 :

Solve $x + 4 = 6$ by a graph.

Let us denote the integers by means of small hollow circles and the solutions by shaded circles.

$$x + 4 = 6$$
$$x + 4 + (-4) = 6 + (-4)$$
$$x = 2$$

Fig. 5-1 gives the solution of $x + 4 = 6$

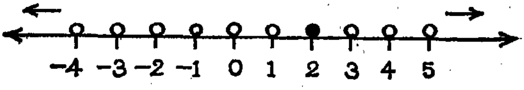


Fig. 5-1.

Example 2 :

Solve $x + 6 = 4$ by a graph.

$$x + 6 = 4$$
$$x + 6 + (-6) = 4 + (-6)$$
$$x = -2.$$

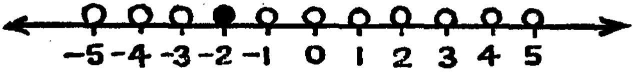


Fig. 5-2.

Exercise 1—1

Solve the following using integers through a graph :

(1) $x + 4 = 4$ (2) $x + 5 = 9$ (3) $x + 8 = 2$

(4) $x + 9 = 5$ (5) $x + 5 = 4$

1—2. One Dimensional Graphs — Inequations $[x + a > b]$

Example 1 :

Solve $x + 4 > 6$ by means of a graph.

$$x + 4 > 6$$

$$x + 4 + (-4) > 6 + (-4)$$

$$x > 2$$

∴ Solutions are the integers greater than 2, i.e., all integers to the right of 2 in the number line.

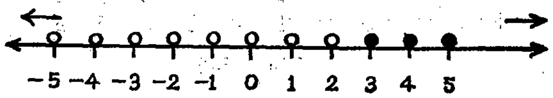


Fig. 5-3.

Example 2 :

Solve $x + 6 > 4$ by means of a graph.

$$x + 6 > 4$$

$$x + 6 + (-6) > 4 + (-6)$$

$$x > -2$$

∴ Solutions are the integers greater than − 2.

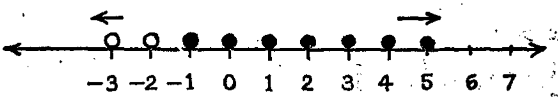


Fig. 5-4.

The integers to the right of − 2 in the number line give he solution.

Exercise 1—2

Using integers solve the following graphically.

(1) $x + 2 > 2$ (2) $x + 2 > 8$

(3) $x + 8 > 3$ (4) $x + 5 > 2$

1—3. One Dimensional Graphs [$ax = b$]

Example :

Solve $4x = 12$ by means of a graph.

$$4x = 12$$

$$\frac{1}{4} \times 4x = \frac{1}{4} \times 12$$

$$x = 3$$

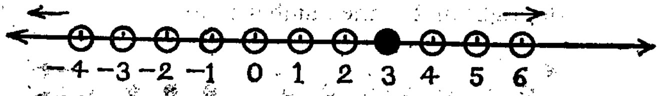


Fig. 5-5.

Exercise 1—3

Find the solutions in integers. Draw also graphs.

(1) $3x = 9$ (2) $4x = 20$

(3) $2x = 0$ (4) $5x = 30$

1—4. To find the solution using fractional numbers

Example :

Draw a graph to solve $2x > 5$ using integers.

$$2x > 5$$

$$\frac{1}{2} \times 2x > \frac{1}{2} \times 5$$

$$x > \frac{5}{2}$$

$$x > 2\frac{1}{2}$$

The solutions are fractional numbers greater than $2\frac{1}{2}$, that is, all numbers to the right of $2\frac{1}{2}$ in the number line. We show this by a dotted line starting from the hollow dot at $2\frac{1}{2}$.

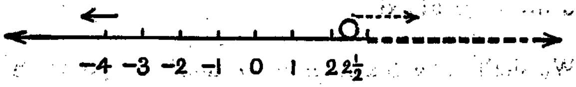


Fig. 5-6.

Exercise 1—4

Solve using fractional numbers by a graph:

(1) $3x > 6$ (2) $2x > 1$

(3) $5x > 0$ (4) $3x > 12$

(5) $2x > 7$

2—1. Two Dimensional Graphs

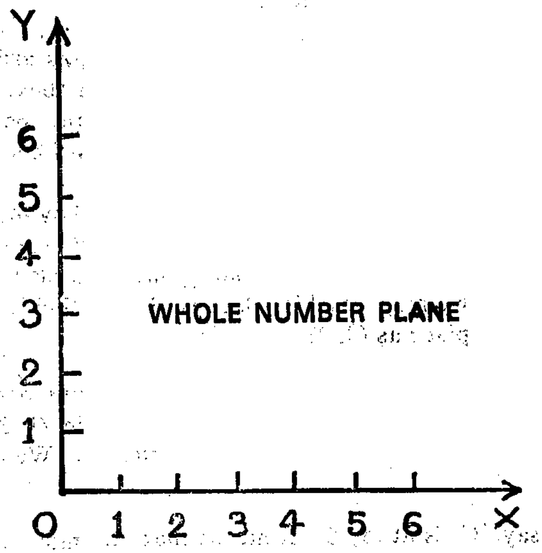


Fig. 5-7.

In the figure, $\overrightarrow{OX}$ is a number ray. $\overrightarrow{OY}$ is another number ray perpendicular to $\overrightarrow{OX}$ through O. O is the common starting point to both $\overrightarrow{OX}$ and $\overrightarrow{OY}$. These two rays form a whole number plane. $\overrightarrow{OX}$ is called the horizontal axis and $\overrightarrow{OY}$ is called the vertical axis.

We shall now learn how to mark a point in a whole number plane.

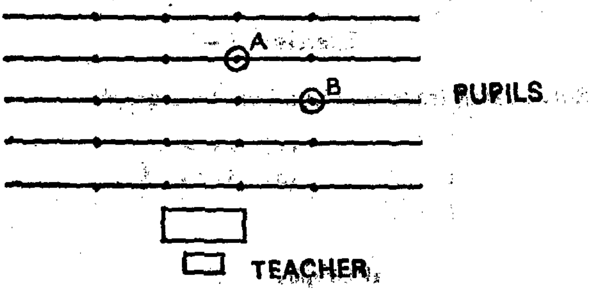


Fig. 5-8.

Here is a diagram of a class room. 20 pupils are seated as above in the class. Each dot denotes a boy. How does the teacher denote the place where the boy A is sitting. The boy is sitting as the 3rd boy in the 4th bench.

Similarly we see that B is sitting as the 4th boy in the 3rd bench. With the help of two measurements, we may denote any point in the whole number plane. Since *A's* place is 3rd in the 4th row, we may say his place is (3, 4). Similarly, we may denote *B's* place as (4, 3).

Let us now mark these points in the whole number plane in Fig. 5-9 For marking (3, 4), we move from O, 3 units horizontally and then move 4 units up vertically. We mark A (3, 4).

If we say, C is at (5, 6), it means that, we move 5 units from O horizontally and go 6 units up vertically.

For denoting $D(0, 3)$, we do not move horizontally but move 3 units up vertically. To mark $E(4, 0)$ move 4 units along $\overrightarrow{OX}$ and do not move in the vertical direction.

The above points $A, B, C, D, E,$ are marked in the squared sheet below. O is called the origin. Its position is (0, 0).

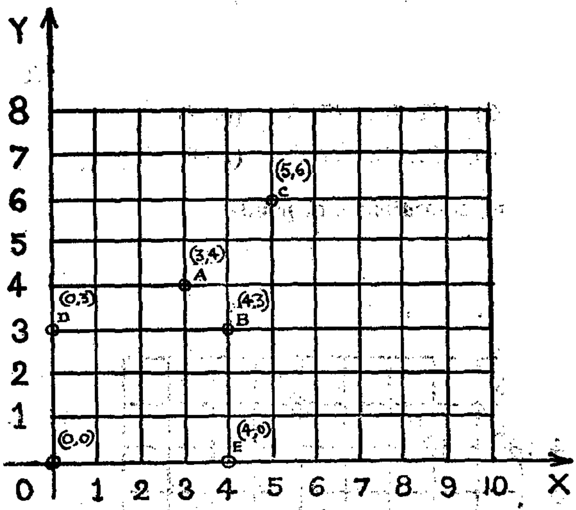


Fig. 5-9.

$\overrightarrow{OX}$, $\overrightarrow{OY}$ are called X axis and Y axis respectively. The two numbers within the brackets to denote a point in a whole number plane are called coordinates. The first number is called the X coordinate, since it tells us how far we go along $\overrightarrow{OX}$. The second number is called the Y coordinate, since it tells us how high we go along $\overrightarrow{OY}$.

Exercise 2—1

1. Find on which axis do the following points lie:

(1) (5, 0) (2) (0, 2)

(3) (0, 6) (4) (4, 0)

2. Do the following represent the same point ?

(1) (1, 3), (3, 1)
(2) (4, 3), (3, 4)
(3) (5, 2), (2, 5)
(4) (4, 0), (0, 4)

3. Plot the following points in a squared sheet :

(1) (2, 2), (4, 4), (6, 6), (7, 7), (0, 0)
(2) (4, 1), (1, 4), (5, 7), (7, 5), (0, 6), (6, 0)
(3) (0, 2), (2, 0), (7, 0), (0, 7)
(4) (6, 4), (5, 2), (1, 7), (2, 8), (8, 2)

2—2. Finding co-ordinates of points

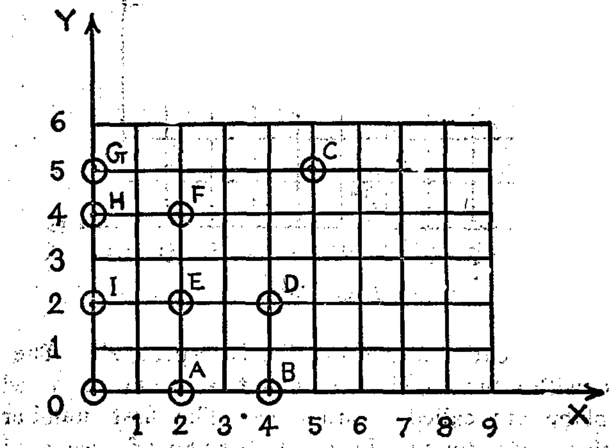


Fig. 5-10.

Coordinates of O are (0, 0)
Coordinates of A are (2, 0)
Coordinates of B are (4, 0)
Coordinates of C are (5, 5)
Coordinates of D are (4, 2)
Coordinates of E are (2, 2)
Coordinates of F are (2, 4)

Coordinates of G are (0, 5)

Coordinates of H are (0, 4)

Coordinates of I are (0, 2)

Exercise 2—2

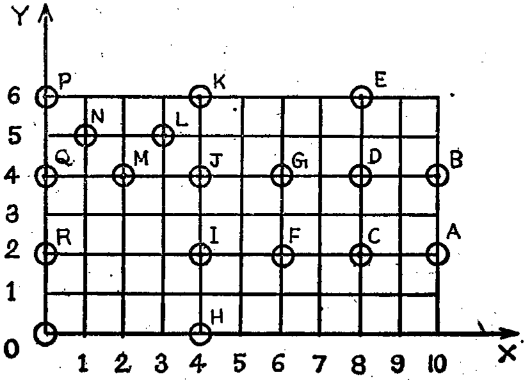


Fig. 5-11.

Write the coordinates of the points A, B, C, ... R in the above graph.

TEST PAPER 5

[Time 1 hour

1. Find the solutions of the following in integers in a graph:

(1) $x+2=8$ (2) $x+8=2$ (3) $x+3=3$
(4) $x+5=6$ (5) $x+7=11$

2. Find the solutions of the following in integers in a graph:

(1) $x+6>5$ (2) $x+6>2$ (3) $x+2>1$
(4) $x+9>4$ (5) $x+1>11$

3. Find the solutions of the following in fractional numbers in a graph:

(1) $4x=13$ (2) $2x=11$ (3) $3x=15$
(4) $5x=8$

4. Find the solutions of the following in fractional numbers in a graph :

(1) $2x > 9$ (2) $3x > 9$ (3) $4x > 11$

(4) $2x > 17$.

5. Plot the points in a squared sheet :

(1) (0, 0)	(2) (6, 7)	(3) (7, 6)
(4) (4, 0)	(5) (0, 4)	(6) (7, 7)
(7) (9, 11)	(8) (10, 13)	(9) (14, 21)
(10) (14, 9)	(11) (10, 2)	(12) (0, 20)

6. Write the coordinates of the points shown in the given graph.

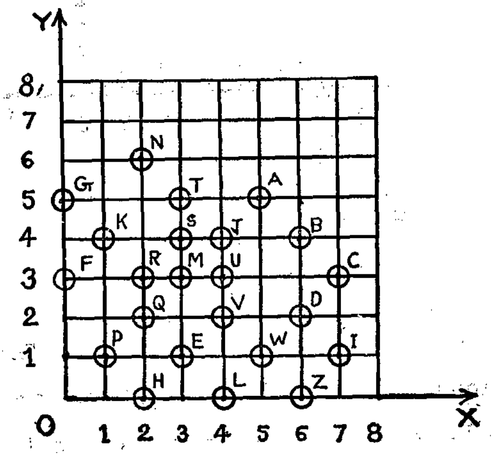


Fig. 5-12.

6. MENSURATION

1. Area

PRE - TEST

Exercise 1—1

1. How many square metres are there in 3 ares ?

2. How many square metres make two hectares? How many ares make 2 hectares ?

3. The length and breadth of a rectangle are 7 m and 50 dm respectively. What is its perimeter ?

4. Rewrite 0·95 ares in square metres.

5. The length, breadth and height of a room are respectively 4 m, 3 m and 3 m. Find the area of the four walls.

6. The length and breadth of a rectangular field are 50 m and 20 m respectively. Find the area in ares.

7. Find the area:

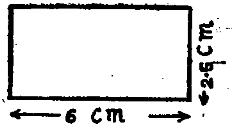


Fig. 6-1.

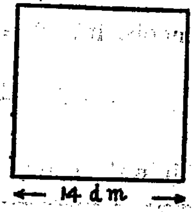


Fig. 6-2.

8. The length and breadth of a room are respectively 6 m and 5 m. Find how many marble stones of square shape whose edge is $\frac{1}{2}$ m long are required for its floor.

9. The side of a square field is 300 m. Give the area in hectares.

10. The area of rectangular field is 45 ares. If its length is 90 m find its breadth.

11. What is the cost of painting a steel plate of length 8 dm 4 cm and breadth 5 dm 2 cm at the rate of 2 paise per square centimetre?

12. The perimeter of a rectangular field is 122 m. Its breadth is 25 m. Find its area.

Practical work :

Measure and find the area of your school ground, garden, class rooms, verandahs, doors and windows

2—1. Area of Rectangle and Square

We have already learnt the formulae for finding the area of rectangles and squares in VI standard.

Area of a rectangle = length × breadth.

In symbols, we write it as $A = l \times b$

Perimeter of a rectangle = 2 (length) + 2 (breadth)

In symbols, we write it as $P = 2l + 2b = 2(l + b)$

Area of a square = side × side = side²

In symbols, it is $A = a \times a = a^2$

Perimeter of a square = side × 4

In symbols, it is $P = 4a$

Exercise 2—1

Oral :

1. Fill in the blanks :

S. No.	Length	Breadth	Area	Perimeter
1	12 cm	10 cm	—	—
2	40 m	—	1200 sq. m	—
3	—	100 m	—	600 m
4	—	40 m	2000 sq. m	—

2. The perimeter of a square is 40 cm. Find its area.

3. The length of a rectangle is x m and its breadth is y m. What is its area ?

4. The side of a square is 14 dam. Find its area and perimetei

Written :

5. The area of a rectangular field is 4·2 hectares. Its length is 300 m. Find its breadth and perimeter.

6. The side of a square field is 60 m. The length of a rectangular field whose area is equal to that of the square field is 90 m. What is its breadth ?

7. The perimeter of a rectangular field is 28 dam. If its breadth is 60 m, find its area in ares.

8. Find the area of the shaded portion in the figure.

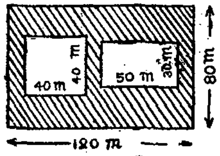


Fig. 6-3.

9. The length and breadth of a road are 1 km and 20 m respectively. How many concrete slabs of length 4 m and breadth $2\frac{1}{2}$ *m are required? Find the cost at the rate of* Rs. 120 per slab.

2—2. Rectangular paths—(Lengthwise or breadthwise — single path)

We have seen paths inside and outside rectangular gardens. The paths may be lengthwise or breadthwise. To go round the garden easily we make cross paths lengthwise and breadthwise or paths on all the four sides. In our rooms also we leave space of equal width on all the four sides and cover

the remaining area with carpet or mat. Let us try to find the area of such paths.

Example 1 :

A room is 8 metres long and 4 metres broad. Outside the room there is a lengthwise verandah of width 2 metres. Find the area of the verandah.

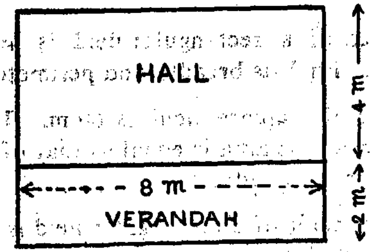


Fig. 6–4.

Length of the verandah = Length of the room = 8 m.

Breadth of the verandah = 2 m.

∴ Area of the verandah = $8 \times 2 = 16$ m²

Example 2 :

A room is 5 metres long and 3·6 metres broad. A stage of width 0·5 metre is put up breadthwise on one side. Find the area of the stage.

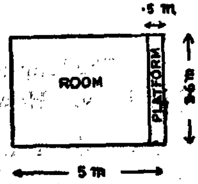


Fig. 6-5.

Length of the stage
= Breadth of the room = 3·6 m.

Breadth of the stage
= 0·5 m.

∴ Area of the stage = $3{\cdot}6 \times 0{\cdot}5$ m²
= 1·80 m² = 1·8 m².

Example 3 :

The length and breadth of a garden are 25 metres and 20 metres respectively. A lengthwise path of width 3 metres runs in the middle of the garden. What is the area of the path?

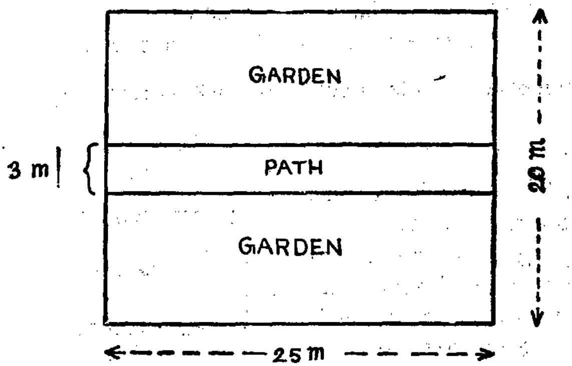


Fig. 6-6.

Length of the path = Length of the garden = 25 m.
Breadth of the path = 3 m.
Area of the path = $25 \times 3\ m^2 = 75\ m^2$.

Exercise 2—2

1. A room is 6 m long and 5 m broad. There is a lengthwise verandah of width 2 m outside the room. Find the area of the verandah.

2. In the above sum if the verandah is put up breadthwise and is of width $1\frac{1}{2}$ m what is its area ?

3. The length and breadth of a calendar are respectively 40 cm and 25 cm. It is attached to a card-board of 2 cm width on the lengthwise side. Find the area of the card-board.

4. The length and breadth of a room are 5 m and 4 m respectively. A concrete slab of width $1\frac{1}{2}$ m is put up breadthwise in it. What is the upper surface area of the slab ?

5. A window is 12 deci metres long and 10 deci metres broad. A wooden frame of width 8 cm is fixed lengthwise in the middle of the window. Find the area of the frame.

6. The length and breadth of an auditorium are 25 metres and 15 metres respectively. A corridor of width 1·6 metres is

reserved lengthwise for movement and seats are arranged in the rest of the area. What is the cost of spreading mat for the corridor at the rate of Rs. 1-50 per square metre ?

2—3. Rectangle — Paths (Lengthwise and breadthwise — Double paths)

Example 1 :

A garden is 40 metres long and 20 metres broad. There are two lengthwise paths of width 1·5 m each adjoining the boundaries inside the garden. What is the total area of the two paths ?

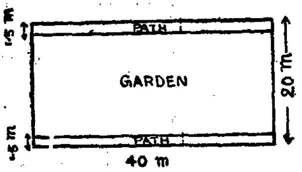


Fig. 6–7.

Length of one path $=$ Length of the garden
$= 40$ m

Breadth of the path $= 1·5$ m

Area of the path $= 40 \times 1·5\ m^2$
$= 60\ m^2$

$\therefore$ Area of the two paths $= 2 \times 60\ m^2$
$= 120\ m^2$.

Example 2 :

There is an open space of breadth 4 m in front of a house. Two cement seats of width 60 cm each are put up on either side in the open space. Find the area of the cement seats.

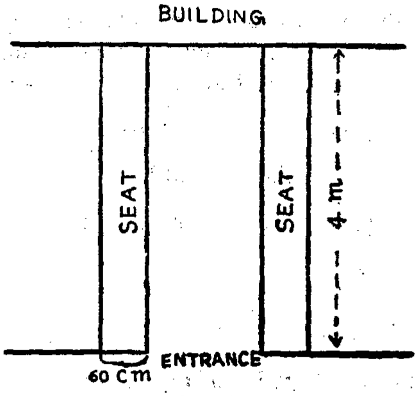


Fig. 6-8.

Length of one seat = Breadth of the open space

= 4 m

Breadth of the seat = 60 cm = ·6 m

Area of one seat = $4 \times \cdot 6\ m^2 = 2\cdot 4\ m^2$

∴ Area of two seats = $2 \times 2\cdot 4\ m^2$

= $4\cdot 8\ m^2$

Exercise 2—3

1. The inner perimeter of a police van is 9 m 60 cm. Its breadth is 1 m 80 cm. If two benches of width 40 cm. are kept lengthwise inside, find the area of the benches and the area of the space inside.

2. The length and breadth of a kitchen are 6 m and 5 m respectively. Two breadthwise platforms are erected, each of breadth 7 dm. What is the total upper surface area of the two platforms ?

3. The length and breadth of a house site are 30 m and 20 m respectively. A house is built after leaving a breadthwise space of width 2 m on both sides. Find the area of the space left.

4. A saree is 5·5 m long and 1·4 m broad. Lengthwise borders of width 6 cm are attached on both sides. Find the total area of the borders.

5. A street in a city is $\frac{1}{4}$ km long and 13 m broad. On either side of the street run platforms, each of width $2\frac{1}{2}$ m. Find the area of both the platforms. Calculate the cost of spreading concrete square slabs of $\frac{1}{2}$ m side on both the platforms at the rate of Rs. 7 per slab.

2—4. Rectangle, Square — (a) Paths running outside — Area

Example :

A rectangular field is 60 m long and 40 m broad. It has pathway 2 m wide all around it. What is the area of the path ?

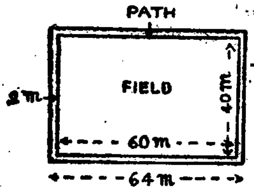


Fig. 6-9.

As the pathway runs all around the rectangular field, the inner edges of the path form a rectangle and the outer edges of the path form another rectangle.

Area of the path = Area of the outer rectangle — Area of the inner rectangle.

Length of the outer rectangle = Length of the inner rectangle + (2 × width of the path)

$$= 60 + (2 \times 2) \text{ m} = (60 + 4) \text{ m} = 64 \text{ m}$$

Breadth of the outer rectangle = Breadth of the inner rectangle + (2 × width of the path)

$$= 40 + (2 \times 2) \text{ m} = (40 + 4) \text{ m} = 44 \text{ m}$$

∴ Area of the outer rectangle $= 64 \times 44 \text{ m}^2$
$= 2816 \text{ m}^2$

Area of the inner rectangle $= 60 \times 40 \text{ m}^2$
$= 2400 \text{ m}^2$

∴ Area of the path $= (2816-2400) \text{ m}^2$
$= 416 \text{ m}^2$

(b) Paths running inside —Area

Example 1 :

The length and breadth of a hall are respectively 8 m and 6 m. Inside the hall red cement is painted to a width of $\frac{1}{2}$ m adjacent to the walls. Find the area of the space painted.

Area of the painted space = Area of the outer rectangle— Area of the inner rectangle.

Area of the outer rectangle
$= 8 \times 5 \text{ m}^2 = 40 \text{ m}^2$

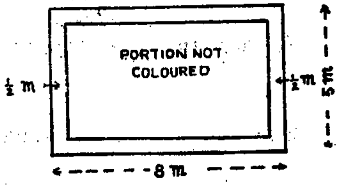


Fig. 6-10.

Length of the inner rectangle = Length of the outer rectangle — (2 × width of the painted space).

$$= 8 - \left(2 \times \frac{1}{2}\right) \text{m} = (8 - 1) \text{ m} = 7 \text{ m}.$$

Breadth of the inner rectangle = Breadth of the outer rectangle — (2 × width of the painted space).

$$= 5 - \left(2 \times \frac{1}{2}\right) \text{m} = (5 - 1) \text{ m} = 4 \text{ m}.$$

Area of the inner rectangle $= 7 \times 4 \text{ m}^2 = 28 \text{ m}^2$

Area of the painted space $= (40 - 28) \text{ m}^2 = 12 \text{ m}^2$.

Example 2 :

A gallery 5 m wide was erected inside a square play ground of side 100 m. Find the area of the gallery.

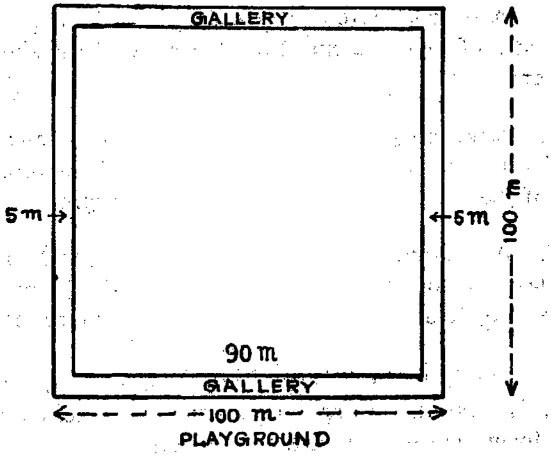


Fig. 6-11.

Breadth of the gallery $= 5$ m

Side of the outer square $= 100$ m

Area of the outer square $= 100 \times 100$ m^2

$= 10000$ m^2

Side of the inner square $=$ Side of the outer square $-$ (2 $\times$ width of the gallery)

$= 100 - (2 \times 5)$ m $= (100 - 10)$ m $= 90$ m

Area of the inner square $= 90 \times 90$ m^2

$= 8100$ m^2

Area of the gallery $= (10000 - 8100)$ m^2

$= 1900$ m^2

Exercise 2 - 4

1. A framed picture is of dimensions 6 dm $\times$ 4 dm. If the frame is 5 cm wide, what is the area of the frame?

2. An entrance door of 2 m $\times$ 1 m has a door frame of 1 dm wide. Find the area of the frame in square decimetres.

3. A framed slate is of dimensions 16 cm $\times$ 12 cm The width of its frame is 2 cm. Find the area of the slate excluding the frame.

4. A wedding invitation card is 18 cm long and 14 cm broad. It has a margin of uniform width 2 cm along the four sides of the card. Find the area of the margin.

5. A rectangular field is 60 m long and 40 m broad. Calculate the cost of paving a path of width 2·5 m all around on the outside at the rate of Rs 3-50 per square metre.

6. The perimeter of an auditorium is 80 m. Its length is 24 m. A carpet is spread on the floor leaving a margin of uniform width 1 m inside the hall. What is the area of (a) the carpet and (b) the auditorium?

7. A garden of length 160 m and breadth 100 m is fenced. A path 2 m wide runs all around the fence on its inner sides. Find the area of the path.

8. In the above problem, calculate the area of the path if the path runs outside the fence.

9. In the middle of a square pond of side 75 m, a square building of side 5 m is put up. If the pond is full of water, what is the surface area of water ?

10. Two paths of width 2 m run all around on the inside and outside of a square lawn of side 20 m. Find the total area of the paths.

2—5. Rectangle, Square — Cross paths — Area

Example :

Two pathways each 2 m wide, one lengthwise and the other breadthwise run across a rectangular plot 60 m long and 40 m wide. Calculate the area of the pathways and the remaining area.

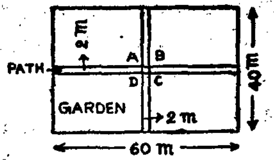


Fig. 6-12.

Area of the garden $= 60 \times 40 \text{ m}^2 = 2400 \text{ m}^2$

Area of the lengthwise path $= 60 \times 2 \text{ m}^2 = 120 \text{ m}^2$

Area of the breadthwise path $= 40 \times 2 \text{ m}^2 = 80 \text{ m}^2$

[The area of the small square (*ABCD*) is common to both the paths and has been included in both the areas of pathways. So, we have to deduct the area of *ABCD*, namely 4 m², from the total area of the paths]

Total area of the paths $= (120 + 80 - 4) \text{ m}^2$

$= (200 - 4) \text{ m}^2 = 196 \text{ m}^2$

The remaining area = Area of the garden — Area of the paths

$= (2400 - 196) \text{ m}^2$

$= 2204 \text{ m}^2$.

Exercise 2—5

1. A rectangular plot is 36 m long and 24 m broad. Two paths 3 metres wide each run across the plot, one lengthwise and the other breadthwise. Find the area of the paths.

2. The perimeter of a square field is 320 m. Two paths of width 2 m each run in the middle of the field, one lengthwise and the other breadthwise. What is the cost of repairing the path at the rate of 50 paise per square metre ?

3. Two pathways, each 5 m wide, one lengthwise and the other breadthwise run across a rectangular garden 120 m long and 80 m. broad. What is the cost of gravelling the paths at the rate of Rs. 1-25 per square metre? The remaining area is levelled at the rate of 20 paise per square metre. What is the total cost ?

4. The inner length and breadth of a window are 1 m 20 cm and 80 cm respectively. It has frames of width 8 cm one running all around and two others in the middle, one lengthwise and the other breadthwise. Find the total area of the frames.

3—1. Triangles — Area

PER—TEST

Exercise 3—1

1. In the following figures name the base and the altitude of the triangles.

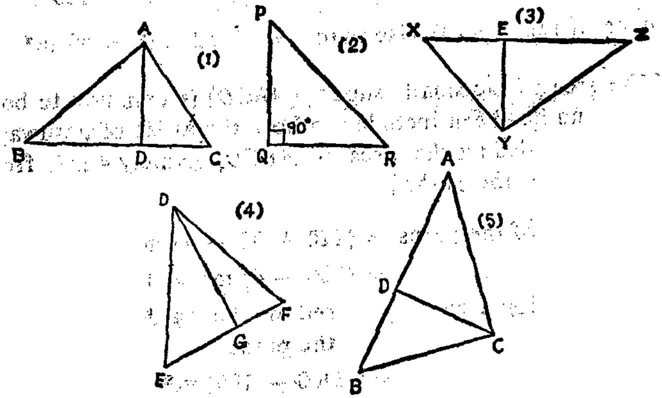


Fig. 6-13.

2. Find the area of the right triangles given below. (The measurements are in centimetres.)

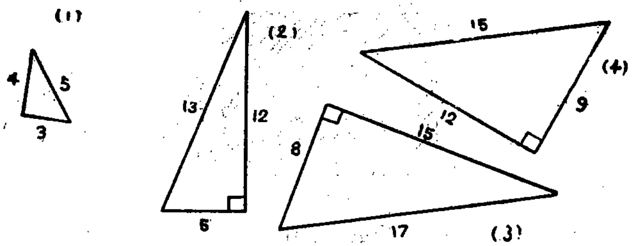


Fig. 6-14.

3. Calculate the area of the following triangles (lengths given in dm).

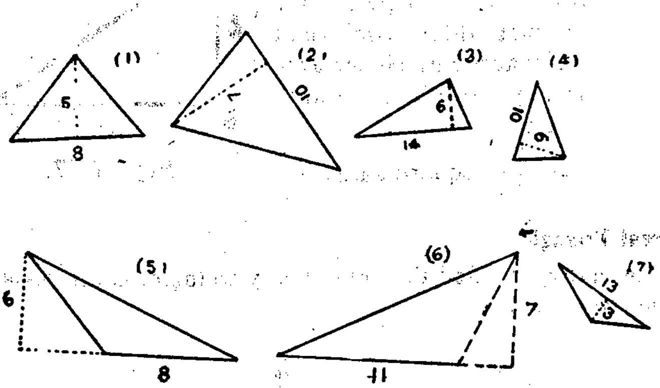


Fig. 6-15.

4. The base of a triangle is 8 cm and its altitude is 3·6 cm. What is its area?

5. The base of a triangular field is 25 m. The shortest distance of the base from the opposite vertex is 16 m. Find the area of the field in ares.

6. The area of a triangular field is 20 cm^2. Its base is 5 cm. Find the corresponding altitude.

7. Find the area of the shaded portion in the figure.

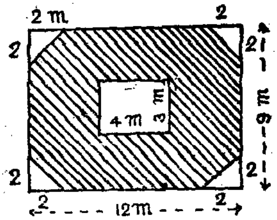


Fig. 6-16.

Right Triangle

When a rectangular sheet of paper is folded along its diagonal, we get two right triangles. We also notice that these two right triangles are congruent. Hence area of a right triangle = half of the area of the rectangle

$= \frac{1}{2}bh$ square units.

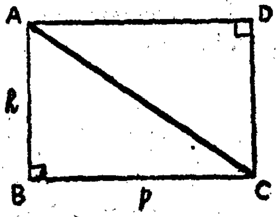


Fig. 6-17.

General Triangle

Let us try to find the area of any triangle, not necessarily a right triangle.

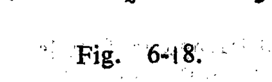


Fig. 6-18.

Cut out a triangle from a piece of paper.

The triangle can be folded such that AD is perpendicular to BC. ΔABC is divided into two right triangles.

$$\text{Area of } \Delta ABC = \text{Area of } \Delta ADB + \text{Area of } \Delta ADC$$

$$= \frac{1}{2} \times BD \times AD + \frac{1}{2} \times DC \times AD \text{ sq. units}$$

$$= \frac{1}{2} \times AD\,(BD + DC) \text{ sq. units}$$

$$= \frac{1}{2} \times AD \times BC \text{ sq. units}$$

$$= \frac{1}{2}\, b\, h \text{ sq. units}$$

'b' is the base of the triangle and 'h' is the altitude from the opposite vertex to the base. In the right triangle the sides containing the right angle form the base and the altitude.

Practical Work

1. Draw some triangles and find their areas.

2. Draw any two triangles in a graph sheet and find the areas by counting the number of small squares. Verify your answer by finding the area using the formula.

(Consider 100 small squares = 1 cm^2. First count the 1 cm^2 squares and then count the small squares. Divide the total number of small squares by 100, it will give the area in cm^2. Add the number of 1 cm^2 squares to this. We get the area of the triangle.)

Note :

While counting the small squares half and more may be considered as whole squares.

4—1. Circle

One of the most important and familiar geometrical figures is a circle. Bangles, Clocks, 1 Rupee coin etc., have got circular shapes; sun and full moon also appear like circles.

What is a circle?

A circle is a figure which contains all the points which are equidistant from a fixed point in a plane. The fixed point is

called the centre of the circle. The fixed distance is known as the radius of the circle. The curved line is known as the perimeter or circumference of the circle.

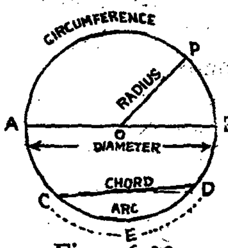


Fig. 6-19.

In *Fig.* 6-19, *O* is the **centre of the circle**. The whole curve is the **perimeter** or **circumference** of the circle. *OP* is the **radius**. *AB* is the **diameter**. The line segment *CD* is the **chord** of the circle. The part *CED* of the circumference is the **arc**.

The line joining the centre and any point on the circumference of the circle is the **radius** of the circle. All the radii of a circle are of equal measure. The line joining any two points on the circumference of the circle is known as the **chord**. *The chord which passes through the centre is the* **diameter** of the circle. Diameter is twice the radius. The length of the arc of a semi circle is the semi perimeter of the circle.

To find the circumference of a circle—Method (1)

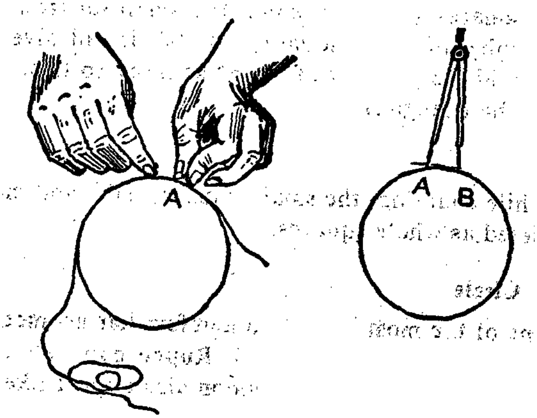


Fig. 6-20.

Ramu drew three circles of radii 2 cm, 3 cm and 4 cm. He calculated the circumference of the circles by measuring

with the help of a thread and dividers as shown in the diagrams and tabulated his results as shown below:

S.No.	Method	Circle No.	Diameter cm.	Perimeter cm.	$\frac{\text{Perimeter}}{\text{Diameter}}$
1	Thread Method	1	4	12·57	3·143
		2	6	18·84	3·140
		3	8	25·14	3·143
2	Dividers Method	1	4	12·56	3·140
		2	6	18·85	3·142
		3	8	25·15	3·144
					18·852

$$\text{The average value of } \frac{\text{Perimeter}}{\text{Diameter}} = \frac{18 \cdot 852}{6}$$
$$= 3 \cdot 14$$

What do you understand from the last column? In all the circles the values of $\frac{\text{Perimeter}}{\text{Diameter}}$ are almost equal. Mathematicians discovered that the ratio of the circumference to the diameter of all the circles is the same. That ratio is denoted by the Greek alphabet π. The perimeter of a circle is approximately 3·14 times its diameter. Another approximate value of π is $\frac{22}{7}$. Both the values 3·14 and $\frac{22}{7}$ are only approximate values of π. It is not possible to find the exact value of π.

Experiment :

Draw five circles of different radii and find the relation between the circumference and diameter of the circles. Check whether your answer is similar to the one above.

Circumference = 3·14 × diameter.

To find the circumference of a circle — Method (2)

If we take articles such as coins, bangles etc which are circles in shape, the circumference can be found by the following methcd.

Mark a point *A* on the circumference of a coin. In a scale mark a point such that *A* coincides with a centimetre mark on the scale. Roll the coin slowly on the scale in a straight line until point *A* coincides again with a point on the scale. Note down the measurement on the scale. The distance between the two points marked on the scale is the circumference of the coin.

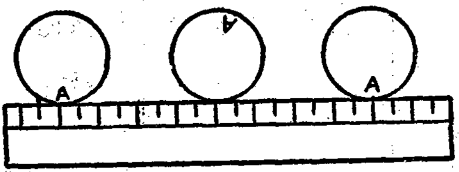


Fig. 6-21.

To find the diameter of a circular article

Place the circular article on a table. Keep two wooden cubes to touch the circular object as shown in the diagram. With the help of the scale find the distance between the two inner edges of the wooden cubes. This is the diameter of the circular article.

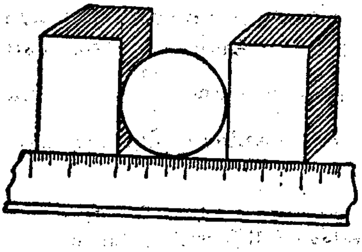


Fig. 6-22.

In the same way find the circumference and diameter of some more circular objects and tabulate the results as follows:

S.No.	Name of the circular object	Length of the circum-ference cm.	Length of the diameter cm.	$\frac{\text{Circumference}}{\text{Diameter}}$

From the table find the relationship between the circumference and diameter of a circle.

The story of Pie :

Mathematicians calculated the relationship between the circumference and diameter of a circle in many ways. The constant value is denoted by the Greek letter π which is to be read as 'pie'.

A Greek Mathematician and Scientist who lived in Sicily, by name Archemedes, discovered the value of π, to be $3\frac{1}{7}$, some 2000 years back.

At first Hebrews took 3 as the value of π. In 1700 B. C. Alms found it to be about 3·1605. In the 7th century B.C. an Indian Mathematician Brahmagupta discovered the value of π as either $\sqrt{10}$ or 3·1623.

In the 5th century B.C. another Indian Mathematician Aryabatta, calculated its value as 3·1416 and the Greek Mathematician Ptolomy took π as 3·14166. The Greek Mathematician Archemedes (287—212 B.C.) found its value to be between $3\frac{1}{7}$ and $3\frac{10}{71}$.

Chang Yeng (75—139 A.D.) discovered its value as $\sqrt{10}$ and Wangfun (229—267 A.D.) calculated it as 3·1555. Another Chinese Mathematician Ivinski calculated it as 3·14. In the 5th century A.D. Chang Chip found it to be between $\frac{22}{7}$ and $\frac{355}{133}$. The great Indian Mathematician Bhaskara (1114—1185 A.D.) found it as 3·1416.

In the 1200 A.D. Leonardo from Pisa calculated the value of π as between 3·140 and 3·1421. Moreover, the value of π as equal to 3, is also mentioned in the New Testament.

In the 20th century, mathematicians calculated the value of π correct to different number of decimal places. We must remember that the value of π has infinite number of decimal places To remember one of the values of π easily, remember the words "Yes, I have a number." The number of letters in each word in order will give the value of π. That is, the value of π is approximately 3·1416.

In general, we take the value of π as $3\frac{1}{7}$ or 3·14. Unless it is given in some other way, we usually take the value of π as $\frac{22}{7}$.

Results obtained from the above facts

In all circles $\frac{\text{Circumference}}{\text{Diameter}} = \pi$. That is, circumference is π times the diameter. If we call circumference as 'C', and diameter as 'd' and the radius as 'r' then

$$C = \pi d \text{ or } C = 2\pi r.$$

Remember :

1. When dividing the circumference by π we get the diameter (d).

2. When dividing the circumference by 2π, we get the radius (r).

Examples :

1. The diameter of a circle is 21 cm. Find its circumference.

$$C = \pi d = \pi \times 21 = \frac{22}{7} \times 21 = 66 \text{ cm.}$$

2. The radius of a circle is 4·2 cm. Find its perimeter.

$$C = 2\pi r = 2\pi \times 4{\cdot}2 = 2 \times \frac{22}{7} \times 4{\cdot}2$$
$$= 26{\cdot}4 \text{ cm.}$$

3. The circumference of a circle is 110 m. What is its diameter ?

$$\pi d = C \qquad \therefore\ d = \frac{C}{\pi} = 110 \times \frac{7}{22} = 35 \text{ m.}$$

4. The circumference of a circle is 132 cm. What is its radius ?

$$2\pi r = C \qquad \therefore\ r = \frac{C}{2\pi} = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm.}$$

Exercise 4—1

Oral :

1. Fill in the blanks suitably :

(1) $\frac{\text{Circumference}}{\text{Diameter}} =$ ______

(2) If we multiply the diameter by π, we get ______ of the circle.

(3) The lengths of all radii of a circle are ______

(4) The greatest chord of a circle is ______

2. (1) The radius of a circle is 3·5 cm. What is the length of the greatest chord that can be drawn in it ?

(2) The radius of a circle is 14 cm. What is its perimeter?

(3) The circumference of a circle is 22 cm. Find its radius.

(4) A and B are any two points on the circumference of a circle. Which is greater, chord AB or arc AB ?

3. (a) Find the circumference of the circles having the following diameters (Give the answer in π) :

(1) 28 m (2) 1 cm (3) 5 cm (4) 14 cm
(5) x m

(b) Find the circumference in terms of π of the circles having the following radii :

(1) $3\frac{1}{2}$ cm (2) 21 cm (3) 4·2 cm
(4) 10·5 cm (5) c cm

(c) The circumference of some circles are given below. Find their diameters.

(1) π cm (2) 7π cm (3) 4π dm
(4) 10π m (5) $x\pi$ m

(d) The circumference of some circles are given below. Find their radii.

(1) 16π cm (2) 3π dm (3) 22π dm
(4) π m (5) $n\pi$ m

Written :

(In the following problems, take $\pi = \frac{22}{7}$)

4. The diameter of a wheel is 25 cm. If it rotates 2800 times what is the distance covered by the wheel. Answer in km.

5. In a clock the length of the minute hand is 3·5 cm. What is the distance the tip of the hand travels in

(a) 1 hour (b) 2 hours ?

6. A wire 154 cm long is bent in the shape of a circle. What is the radius of the circle ?

7. A play-ground is circular in shape. Its perimeter is 2420 m. Find its radius.

8. The diameter of a cart wheel is 70 cm. It rotates 100 times in a minute. What is the speed of the cart in km per hour ?

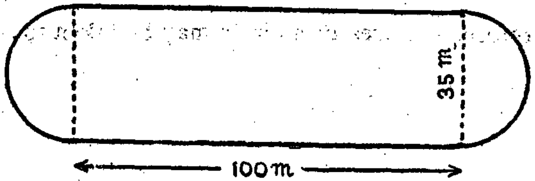
9. 

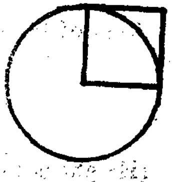
Fig. 6-23.

This is the plan of a play ground. If one goes round it once, what is the distance covered ? What is the cost of fencing the ground at the rate of Rs. 1-50 per metre ?

10. What is the side of a square whose perimeter is equal to the circumference of a circle of radius 70 cm ?

11. Around a circular ground, 20 students were arranged by a teacher each at a distance of $5\frac{1}{2}$ m away from the other. Find the radius of the ground.

4-2. Circle — Area



As shown in Fig. 6-24 draw a circle of radius 1 cm and square of side 1 cm on a graph sheet. Count the number of small squares both in the circle and the square (while counting the square mm, half and more than half of small squares may be taken as full squares.)

Fig. 6-24.

You can find that there are 314 small squares in the circle and 100 small squares in the square. The side of the square is equal to the radius (r) of the circle.

$$\text{Area of the square} = r \times r = r^2$$

$$\frac{\textbf{Area of the circle}}{\textbf{Area of the square}} = \frac{314}{100} = 3{\cdot}14$$

We know that 3·14 is an approximate value of π.

Therefore, the area of a circle may be taken to be πr^2 sq. units.

Aliter :

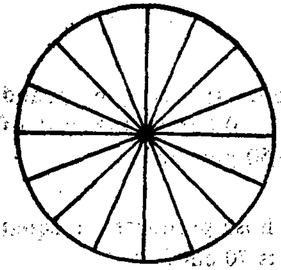


Fig. 6-25.

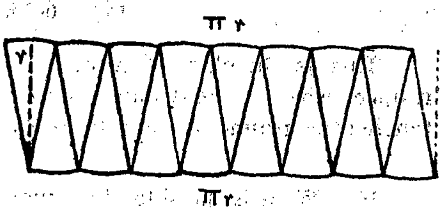


Fig. 6-26.

Divide a circle into 16 equal sectors as shown in Fig. 6-25; cut the sectors and put them together as in Fig. 6-26. Now we get a fiigure like a parallelogram. If the circle is cut into more number of equal sectors, we almost get a rectangle when the sectors are pasted as shown above.

$$\begin{aligned}\text{Area of the circle} &= \text{Area of the parallelogram (rectangle)}\\ &= \text{base} \times \text{height}\\ &= \pi r \times r = \pi r^2 \text{ sq. units.}\end{aligned}$$

(Height is the radius of the circle. Base is half the circumference of the circle, that is πr)

Example 1 :

The radius of a circle is 14 cm. What is its area ?

Radius of the circle (r) $= 14$ cm.

$\therefore$ Area of the circle $= \pi r^2 = \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= 616 \text{ cm}^2$

Example 2 :

The radii of two circles are $3\frac{1}{2}$ cm, $10\frac{1}{2}$ cm. What are the ratios of their circumferences and areas ?

Radius of the first circle (r) $= 3\frac{1}{2}$ cm

Circumference of the first circle $= 2\pi r$

$= 2 \times \pi \times \frac{7}{2} \text{ cm} = 7\pi \text{ cm}$

Area of first circle $= \pi r^2 = \pi \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= \frac{49}{4} \pi \text{ cm}^2$

Radius of the second circle (R) $= 10\frac{1}{2}$ cm

Circumference of the second circle $= 2\pi R$

$= 2 \times \pi \times \frac{21}{2} \text{ cm} = 21\pi \text{ cm}$

Area of the second circle $= \pi R^2$

$= \pi \times \frac{21}{2} \times \frac{21}{2} \text{ cm}^2$

$= \frac{441}{4} \pi \text{ cm}^2$

Ratio of their circumferences $= 7\pi : 21\pi = 1 : 3$

Ratio of their areas $= \frac{49}{4}\pi : \frac{441}{4}\pi = 1 : 9$

Try to know :

1. The ratio of the circumferences of two circles = the ratio of their radii.

$$2\pi r : 2\pi R = r : R$$

2. The ratio of the areas of two circles = the ratio of the squares of their radii.

$$\pi r^2 : \pi R^2 = r^2 : R^2$$

Example 3 :

If we walk twice around a circular ground it is 528 m. Find the area of the ground.

Length of two rounds $= 528$ m.

Circumference of the ground $= \frac{528}{2}$ m $= 264$ m

Radius of the ground $= \frac{C}{2\pi} = \frac{264}{2} \times \frac{7}{22}$ m $= 42$ m

Area of the ground $= \pi r^2 = \frac{22}{7} \times 42 \times 42$ m^2 $= 5544$ m^2

Exercise 4–2

Oral :

1. Find the area of the circles having the following measurements. (Answer in π.)

(a) Radius : (1) 3 cm (2) 1 cm (3) 3·5 cm (4) 20 dm (5) x m

(b) Diameter : (1) 14 cm (2) 10 cm (3) 5 cm (4) 20 dm (5) a m

2. The diameter of a circle is 20 cm. What is its area ? ($\pi = 3{\cdot}14$).

3. The radius of a circle is equal to the side of a square. How many times will the area of the square be equal to the circle ?

4. The side of a square is 14 cm. What is the area of the greatest circle that can be inscribed in it ?

5. What is the area of a circle whose circumference is 44 cm.

Written :

6. The circumference of a circular lawn is 132 m. Find its area in ares.

7. From a square card board of side 28 cm the greatest possible circle is cut out. What is thc remaining area of the card board ?

8.

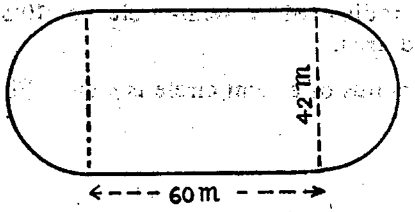


Fig. 6-27.

The plan of a play ground is given. What is the cost of levelling the ground at the rate of Rs. 10 per 100 sq. metres ?

9. There are two equal circles in a rectangle as shown in the diagram. Find the area of the shaded portion.

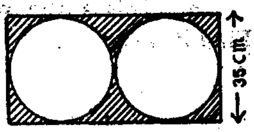


Fig. 6-28.

10. If we run 50 times round a circular ground, it is 3·3 km. Find the area of the ground.

11. Two wires each of length 44 cm are bent in the shape of a square and a circle. Which area is greater and by how much ?

4—3. Semi circle

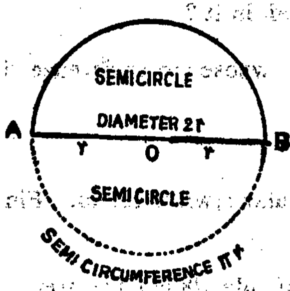


Fig. 6-29.

Draw a circle with centre *O*. Draw the diameter *AOB* in it. We get two semi circles.

Area of the semi circle

$$= \frac{1}{2}\pi r^2 \text{ sq. units.}$$

The perimeter of the semi circle
= Half the circumference +2 radii
$= \pi r + 2r$ units.

Exercise 4—3

1. The radius of a semi-circle is 10·5 cm. Find its perimeter and area.

2. The radius of a semi circle is *a* cm. Find its perimeter and area.

3. This is the plan of a window. Find its area.

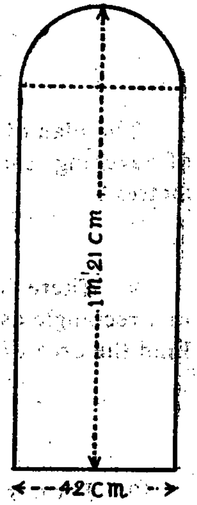


Fig. 6-30.

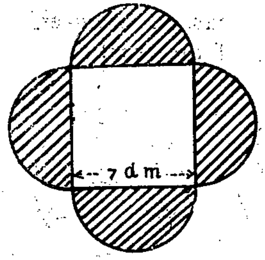


Fig. 6-31.

4. Find the total area of the flower beds (shaded portion) around the square.

5. Find the area of the shaded portions in the two circles.

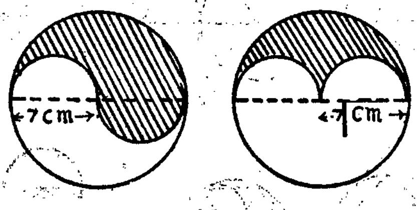


Fig. 6-32.

6. The picture of a kite is given in Fig. 6-33. Find the area of the kite.

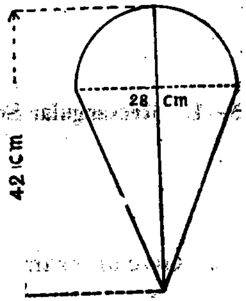


Fig. 6-33.

Practical exercise in drawing circular designs

By using a scale and pair of a compasses, draw enlarged diagrams as shown below with the help of the teacher.

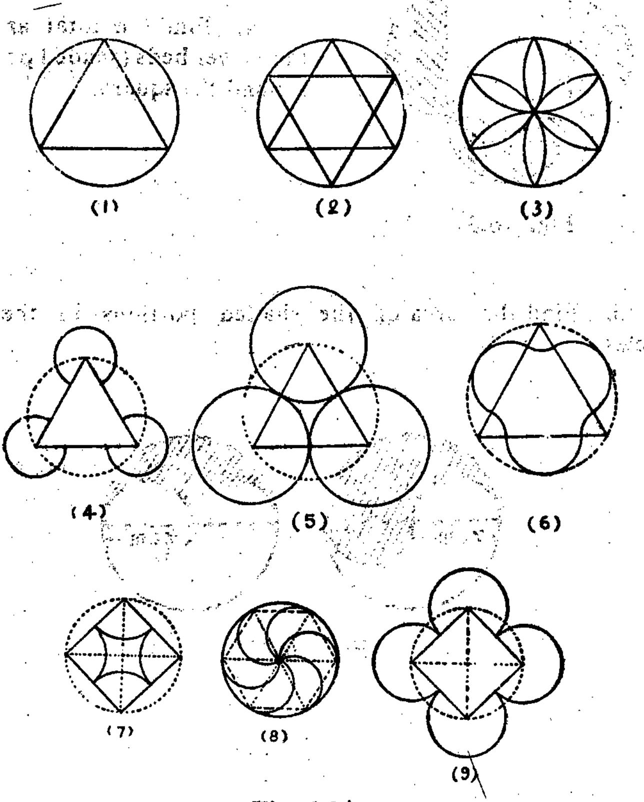


Fig. 6-34.

5—1. Rectangular Solids

PRE — TEST

Exercise 5—1 (a)

1. Give an example each for a cube and a cuboid.
2. Give the formulae for finding the volumes of a cube and a cuboid.

3. What is the formula for finding the area of the four walls of a room?

4. How many cubic decimetres make 1 litre?

5. How many litres are there in 1 cubic metre?

6. How many cubic centimetres make 1 litre?

7. The length, breadth and height of a cuboid are 20 cm, 10 cm and 8 cm respectively. What is its volume?

8. The edge of a cube is 4 cm. Find its volume.

9. The volume of a cube is x^3 dm^3. What is the measure of it edge?

10. The length, breadth and height of a room are respectively 6 m, 5 m and 4 m. Find the area of the four walls.

11. The internal dimensions of a tub are 6 dm $\times$ 4 dm $\times$ 3 dm If it is to be painted inside, what area is to be painted?

Cuboid

We have seen wooden boxes and boxes of tin. How many faces (surfaces) do they have? What is the shape of chalk boxes, soap boxes, match boxes, erasers and bricks?

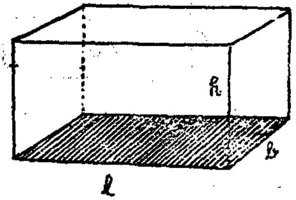


Fig. 6-35.

Let us try to recall some more facts about them.

Let us look at the given figure. How many faces does it have? What do we see about the opposite faces? The four faces at the sides are called the lateral surfaces of the cuboid. If the areas of the top face and the bottom face, i.e. twice the area of the bottom face, are added to the lateral surface area, the total surface area of the cuboid is obtained.

Formula for surface areas

We have already learnt to find the area of the four walls of a room.

Area of the four walls
$= 2 \times \text{height} \times (\text{length} + \text{breadth})$ sq. units.

Area of the bottom face and the top face
$= (\text{length} \times \text{breadth} + \text{length} \times \text{breadth})$ sq. units.

Total surface area

$$= 2 \times \text{height} \times (\text{length} + \text{breadth}) + 2 \times \text{length} \times \text{breadth}$$

$$= 2 \times \text{height} \times \text{length} + 2 \times \text{height} \times \text{breadth} + 2 \times \text{length} \times \text{breadth}$$

$$= 2\,lh + 2\,bh + 2\,lb$$

$$= 2\,(lh + bh + lb) \text{ sq. units}$$

where l = length, b = breadth, h = height

Practical Work :

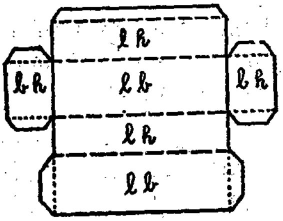


Fig. 6-36.

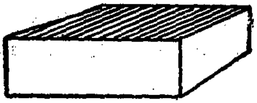


Fig. 6-37.

Take a card board. Draw a figure as shown in Fig. 6-36 and fold the board along the dotted lines. We get a rectangular box.

The area of the 6 rectangles in Fig. 6-36 will give the total surface area of the cuboid in Fig. 6-37.

Cube

This is the picture of a cube. It has 6 congruent square faces A cuboid having all its edges of equal measure is a cube.

Some of the tea packets are in the shape of a cube.

Area of a side = length of the side × length of the side sq. units.

Area of 6 sides = 6 × length of the side × length of the side sq. units

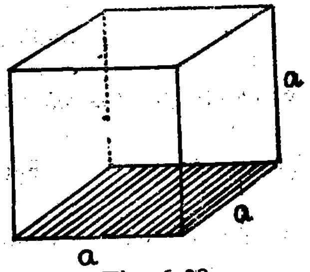


Fig. 6-38.

$= 6 \times a \times a$ sq. units

$= 6\, a^2$ sq. units (Formula)

where a denotes the side of a square.

Practical Work :

Take a thin card board and draw a figure as shown in Fig. 6-39. Proceed as for making a cuboid. We will get a cube.

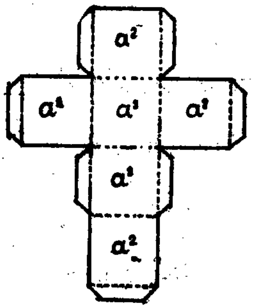


Fig. 6-39.

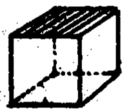


Fig. 6-40.

The six squares drawn in the card board are the six faces of the cube.

Volume of a cuboid and a cube

We have already studied how to find the volume of a cuboid and a cube.

Volume of a cuboid

= length × breadth × height (cubic units).

Symbolically $V = lbh$ cubic units (Formula).

(V = volume; l = length; b = breadth; h = height).

A cube is a cuboid having all its edges of equal measure. That is, $l = b = h$.

Volume of a cube = $l \times l \times l = l^3$ cubic units.

Its volume will be $a \times a \times a = a^3$ cubic units if each of its edges is equal to 'a' units.

Symbolically, $V = a^3$ cubic units (Formula).

Cubic centimetre, cubic metre can be written as cm^3 and m^3 respectively.

Volumes (Table)

1000 cubic millimetre	=	1 cubic centimetre
1000 cubic centimetre	=	1 cubic decimetre
1000 cubic decimetre	=	1 cubic metre
1000 cubic metre	=	1 cubic deca metre
1000 cubic deca metre	=	1 cubic hecta metre
1000 cubic hecta metre	=	1 cubic kilo metre.

Capacity :

10 milli litres	=	1 centi litre
10 centi litres	=	1 deci litre
10 deci litres	=	1 litre
10 litres	=	1 deca litre
10 deca litres	=	1 hecta litre
10 hecta litres	=	1 kilo litre.

Some common measures in use :

1 cubic centi metre	=	1 milli litre
1000 milli litres	=	1 litre
1000 cubic centimetres or 1 cubic deci metre	=	1 litre.
1 cubic metre	=	1000 litres.

When dimensions are given like 20 m × 15 m × 10 m, the first dimension represents the length, the 2nd the breadth and the 3rd the height.

Example 1 :

The external dimensions of a rectangular box are 18 cm × 10 cm × 8 cm. What is its total surface area ?

Length = 18 cm; breadth = 10 cm; height = 8 cm.

Area of four faces = 2 × height × (length + breadth)

$= 2 \times 8 \times (18 + 10)\ cm^2$

$= 16 \times 28\ cm^2 = 448\ cm^2$

The area of top and bottom faces

= 2 × length × breadth

$= 2 \times 18 \times 10\ cm^2 = 360\ cm^2$

Total surface area

$= (448 + 360)\ cm^2 = 808\ cm^2$

Note :

For an open box, we do not take into account the area of the top surface.

If, in the above problem, the box has no lid, then,

Total surface area

$$= 2 \times h \times (l + b) + lb$$

$$= 2 \times 8 \times (18 + 10) + 18 \times 10 \text{ cm}^2$$

$$= 16 \times 28 + 18 \times 10 \text{ cm}^2$$

$$= (448 + 180) \text{ cm}^2 = 628 \text{ cm}^2.$$

Example 2 :

Find the total surface area of a closed cubical box whose side is 12 cm. All the faces are congruent.

Area of 1 face = $12 \times 12 \text{ cm}^2 = 144 \text{ cm}^2$

Area of 6 faces = $144 \times 6 \text{ cm}^2 = 864 \text{ cm}^2$

$\therefore$ Total surface area $= 864 \text{ cm}^2$

Using the formula, total surface area of a box in the shape of a cube = $6a^2$ (a-side)

$$= 6 \times 12 \times 12 \text{ cm}^2 = 864 \text{ cm}^2.$$

Example 3 :

The dimensions of a class room are 10m × 6m × 4m. If a student needs an area of 2 sq. m, how many boys can be accommodated ? How many cubic metres of air does each boy get ?

Area of the room $= 10 \times 6 \text{ m}^2 = 60 \text{ m}^2$

No. of students accommodated $= \frac{60}{2} = 30$ students

Volume of air in the room $= lbh$

$$= 10 \times 6 \times 4 \text{ m}^3$$

$$= 240 \text{ m}^3$$

Volume of air for 30 students = 240 m^3

Volume of air that a student gets $= \frac{240}{30} = 8\ m^3$.

Example 4 :

The length and breadth of a sports arena are 30 m and 10 m respectively. What is the total cost for raising the level of the ground by 20 cm if it costs Rs. 1-50 per cubic metre.

Volume of sand needed to be spread

$= lbh$ cubic units

$= 30\ m \times 10\ m \times 20\ cm$

$= 30 \times 10 \times \frac{20}{100}\ m^3$

$= 60\ m^3$

Cost of raising the level of the ground at Rs. 1-50 per m^3

= Rs. 1-50 × 60

= Rs. 90.

Exercise 5—1 (b)

Oral :

1. Find the total surface area and volume of the cubes whose measures of edges are given below.

(1) 4 cm (2) 10 dm (3) 2 m
(4) 8 cm (5) x m

2. Calculate the total surface area of the cuboids whose measures of edges are given below. :

	Length	Breadth	Height
(1)	8 cm.	6 cm.	5 cm.
(2)	1 m.	10 cm.	2 cm.
(3)	6 cm.	4 cm.	$2\frac{1}{2}$ cm.
(4)	8 dm.	$6\frac{1}{2}$ dm.	4 dm.
(5)	a m.	b m.	c m.
(6)	20 cm.	15 cm.	10 cm.

Written :

3. Calculate the total surface area of the following closed boxes :

	Length	Breadth	Height
(1)	12 dm.	7 dm.	$4\frac{1}{2}$ dm.
(2)	6 dm.	4 dm.	20 cm.
(3)	1·5 m.	6 dm.	15 cm.
(4)	8 dm.	5 dm.	25 cm.

4. The internal measurements of an open wooden box are 24 cm × 16 cm × 12 cm Find the area of the paper pasted both inside and outside the box.

5. The length, breadth and height of a Kalyana Mantapam are 12 m, 8 m and 5 m respectively. What is the total cost of painting the four walls and the ceiling of the Mantapam at the rate of 50 p. per m^2? Deduct the area of the doors and windows which comes to 34 m^2.

6. With the available tin sheets, 8 closed boxes of size 20 cm × 20 cm × 30 cm can be made. With the same tin sheets, how many closed boxes of size 4 cm × 4 cm × 8 cm can be made ?

7. Find how many bricks of size 12 cm × 8 cm × 4 cm are needed to build a wall of size 6 m × 40 cm × 3 m ?

8. The volume of a water tub is 720 liters. The internal length and breadth are 1 m 5 dm and 1 m 2 dm. What is its depth ?

9. The internal measurements of a box are 60 dm × 40 dm × 20 dm. Calculate how many soap boxes of size 10 cm × 4 cm × 2 cm can be kept inside ?

10. How many wooden cubes of edge 2 cm can be cut from a wooden cube of edge $\frac{1}{2}$ m ?

11. Here is the diagram of a victory stand. Find its volume.

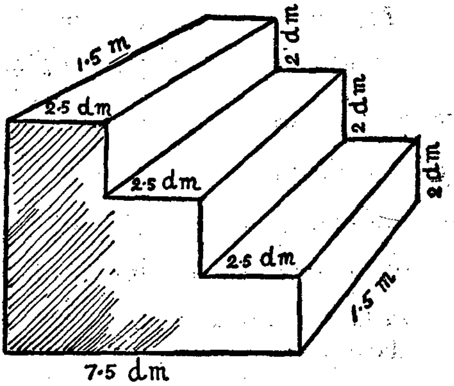


Fig. 6-41.

TEST PAPER 6 (1)

[Time : 45 minutes]

1. Fill in the blanks suitably :

(a) The edge of a cube is 4 cm. Its volume is ———

(b) The length, breadth and height of a room are 6m, 5 m and 3 m respectively. The volume of air inside the room is ———

(c) The perimeter of a square is 80 m. Its area is ———

(d) The diameter of a semi circle is 14 cm. Its area is ———

(e) The radius of a circle is 10·5 cm. The length of the largest chord that can be drawn inside the circle is ———

(f) The perimeter of a rectangle is 40 dm. Its length is 15 dm. Its area is ———

2. Match the following :

(1)	Area of a rectangle	(a)	$2h(l+b)$
(2)	Area of a circle	(b)	$2\pi r$
(3)	Perimeter of a square	(c)	$2(lb+lh+bh)$
(4)	Area of four walls	(d)	lb
(5)	Area of a triangle	(e)	$4a$
(6)	Area of a square	(f)	πr^2
(7)	Circumference of a circle	(g)	a^2
(8)	Total surface area of a cuboid	(h)	$\frac{1}{2}bh$

3. The edge of a square wooden piece is 28 cm. From it the largest possible circular disc is cut out. Find the area of the wastage of wood. $\left(\pi = \frac{22}{7}\right)$

4. The length, breadth and height of an open tin box are respectively 12 cm, 10 cm and 8 cm. Find the area of the metal needed to make it.

5. The length, breadth and height of a fish tank are 40 cm, 30 cm and 20 cm respectively. Find how many litres of water can be poured into it.

6. A rectangular park is of size 80 m × 50 m. Find the area of the path of width $2\frac{1}{2}$ m running round inside the park.

7. (a) Find the difference between the area of a square of side 9 cm and of area 9 cm^2.

(b) Find the area of a circle whose diameter is 2 cm.

8. Find the volume, if 20 books of size 18 cm × 12 cm × 1 cm are kept one above the other.

TEST PAPER 6 (2)

[*Time :* 1 *hour.*]

1. In a dining hall of size 10 m × 6 m five rows of mats of width 40 cm each are spread breadthwise. What is the total area covered by the mats ?

2. A path of width 3 m runs round outside a garden whose length is 84 m and breadth 56 m. What is the cost for levelling the path at the rate of 50 p. per square metre ?

3. The two sides that contain a right angle in a triangle are of lengths 7 cm and 8 cm. Find the area of the triangle ?

4. The length of a side of a rectangular field is 40 m. The shortest distance from the opposite vertex to it is 15 m. Find its area in ares.

5. The circumference of a circle is 44 cm. What is its radius and area ?

6. A circular water tank of radius 3·5 m is placed in the middle of a square lawn, whose side is 16 m. Find the area of the lawn excluding the water tank.

7. A cow is tied to a peg by a rope of 7 m long. What is the area that the cow can graze?

8. The cross section of an almirah is a rectangle with a semicircle on its top along the breadth. The length and breadth of the rectangular section are 1 m 20 cm and 84 cm respectively. If the almirah has two doors of equal size, what is the area of each door ?

ANSWERS

Exercise 1—1

(1) 300 m^2 (2) 20,000 m^2, 200 ares (3) 24 m
(4) 95 m^2 (5) 42 m^2 (6) 10 ares (7) (i) 15 cm^2
(ii) 196 dm^2 (8) 120 stones (9) 9 hectares
(10) 50 m (11) Rs. 87.36 (12) 900 m^2

Exercise 2–1

(5) 140 m, 800 m (6) 40 m (7) 48 ares
(8) 6500 m^2 (9) 2000 slabs; Rs. 2,40,000/-.

Exercise 2—2

(1) 12 m^2 (2) 7·5 m^2 (3) 80 cm^2
(4) 6 m^2 (5) 960 cm^2 (6) Rs. 60

Exercise 2—3

(1) 2·4 m^2; 3·0 m^2 (2) 700 dm^2 (3) 80 m^2
(4) 6600 cm^2 (5) 1250 m^2 Rs. 35,000

Exercise 2—4

(5) Rs. 1837·50 (6) 308 m^2; 384 m^2 (7) 1024 m^2
(8) 1056 m^2 (9) 5600 m^2 (10) 320 m^2

Exercise 2—5

(1) 171 m^2 (2) 316 m^2; Rs. 158 (3) Rs. 1218·75; Rs. 2943·75 (4) 4992 cm^2

Exercise 3—1

(1) $b = BC$, $h = AD$; $b = QR$, $h = PQ$; $b = XZ$, $h = YE$; $b = EF$, $h = DG$; $b = AB$, $h = CD$

(2) 6 cm^2, 30 cm^2, 60 cm^2, 54 cm^2

(3) 20 dm^2; 35 dm^2; 42 dm^2; 30 dm^2; 24 dm^2; 38·5 dm^2; 19·5 dm^2

(4) 14·4 cm^2 (5) 2 ares (6) 8 cm (7) 88 m^2

Exercise 4—1

(4) 2·2 km (5) i. 22 cm ii. 44 cm (6) 24·5 cm
(7) 385 m (8) 13·2 km (9) 310 m; Rs. 465
(10) 110 m (11) 17·5 m

Exercise 4—2

(6) 13·86 ares (7) 168 cm^2 (8) Rs. 390·60
(9) 525 cm^2 (10) 346·5 m^2 (11) circle; 33 cm^2

Exercise 4—3

(1) Perimeter : 54 cm ; Area : $173 \cdot 25$ cm^2

(2) $\pi a + 2a$; $\frac{\pi a^2}{2}$ (3) 5775 cm^2 (4) 77 dcm^2

(5) (i) 77 cm^2 (ii) $38 \cdot 5$ cm^2 (6) 700 cm^2

Exercise 5—1 (a)

(2) cube : a^3; cuboid : lbh (3) $2h(l + b)$
(4) 1 dm^3 (5) 1000 litres (6) 1000 cm^3
(7) 1600 cm^3 (8) 64 cm^3 (9) x dm
(10) 88 m^2 (11) 84 dm^2.

Exercise 5—1 (b)

(3) (i) 339 dm^2 (ii) 448 dm^2 (iii) 243 dm^2
(iv) 145 dm^2
(4) 2688 cm^2 (5) Rs. 131 (6) 160
(7) 18,750 (8) 4 dm (9) 6,00,000
(10) 15,625 (11) 450 dm^3

TEST PAPER 6 (1)

1. (a) 64 cm^3 (b) 90 m^3 (c) 400 m^2
(d) 77 cm^2 (e) 21 cm (f) 75 dm^2

2. (1) lb (2) πr^2 (3) $4a$ (4) $2h(l + b)$
(5) $\frac{1}{2} b \times h$ (6) a^2 (7) $2\pi r$
(8) $2(lb + lh + bh)$

3. 168 cm^2 4. 472 cm^2 5. 24 litres. 6. 625 m^2

7. (a) 72 cm^2 (b) $\frac{22}{7}$ cm^2 8. 4320 cm^3

TEST PAPER 6 (2)

(1) 12m^2 (2) Rs. 438 (3) 28 cm^2 (4) 3 ares
(5) Radius : 7 cm; Area : 154 cm^2 (6) $217 \cdot 5$ m^2
(7) 154 m^2 (8) 6426 cm^2

7. APPLICATION

1—1. Ratio and Proportion

PRE—TEST

Exercise 1—1

1. State as a ratio :

(a) Re. 1; 30 ps. (b) 6 l ; 400 ml

(c) 1 kg ; 250 g. (d) 5 cm ; 10 mm

(e) 3 sq. m ; 24 sq. dm. (f) 1 hr ; 10 mts

(g) 3 dozens; 5 scores. (h) Father's age is 70; Son's age is 42.

2. Write one equivalent ratio for each of the following :

(a) 2 : 7 (b) 1 : 3 (c) 5 : 3

(d) 7 : 9 (e) $\frac{1}{4} : 1$ (f) 1 : 5 : 6

3. Find pairs of equivalent ratios in the following :

(a) 9 : 8 (b) 3 : 4 (c) 15 : 16

(d) 4 : 5 (e) 30 : 32 (f) 18 : 16

(g) 15 : 20 (h) 20 : 25

4. Express the following as a fraction :

(1) 2 : 7 (2) 11 : 15 (3) 7 : 16 (4) 8 : 12

5. Express the following as ratios :

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $\frac{1}{7}$ (4) $\frac{4}{9}$

6. The measures of the angles of a triangle are in the ratio 1 : 2 : 3. Find the measure of the greatest of the three angles.

7. A sum of Rs. 350 is divided between *A* and *B* in the ratio 7 : 3. How much does each get ?

8. Malaravan is 14 years old and his mother is 36 years old. Find the ratio of their ages, (i) before 3 years; (ii) after 3 years.

9. 120 note-books are sold in a school Co-operative stores for Rs. 180. What is the cost of 90 note books ?

1—2. Drawing to scale

It is not possible to draw on a sheet of paper the pictures of a school, house, play ground or our country with the actual measurements. We can only draw smaller pictures to represent them.

Such small pictures actually represent the original piece, just like your photograph gives a picture of yours. Let us take you are 100 cm tall and your photograph shows you 10 cm tall. Every part of your body will be shown in the photograph one-tenth of its actual size. For example, if your face is 20 cm. long, in the photograph it will be 2 cm long. The ratio between the measurements in the picture and the original is called the scale. In the photograph, we say the scale is 1 : 10. It means 1 cm in the photograph represents 10 cm. of the original. It is also sometimes written as 1 cm = 10 cm If it is given as a fraction $\frac{1}{10}$, it is called a 'representative fraction' and in short it is denoted as *R. F.*

Similarly geographical maps are also drawn to scale.

Example 1 :

1 cm represents 1 dam in a map. What is the scale used? What is the representative fraction ?

1 dam = 1000 cm.

1 cm. represents 1 dam or 1000 cm.

$\therefore$ Scale = 1 : 1000

Representative fraction = $\frac{1}{1000}$.

Example 2 :

The representative fraction of a map is $\frac{1}{10000}$. What actual distance will be represented by 1 cm in the map ?

Scale = 1 : 10000

1 cm in the map represents 10000 cm or 100 m in the original.

Example 3 :

The actual distance between Madras and Coimbatore is 480 km. The distance is represented by 1·2 cm in a map. What is the scale ?

1·2 cm represents 480 km.

1 cm represents $\frac{480}{1\cdot2}$ km or 400 km.

∴ The scale : 1 cm = 400 km.

Exercise 1—2

1. Fill in the blanks suitably :

Sl. No.	Scale	Actual length	Length as in the picture
1.	1 cm = 500 m	—	3 cm
2.	1 cm = 200 m	1 km	—
3.	1 cm = —	300 km	2 cm
4.	1 cm = 1 dam.	35 m	—
5.	1 cm = 800 m	4 km	—

2. Find out suitable scales for the following so that the line segment to be drawn does not exceed 8 cm.

(a) 200 m (b) .50 dm (c) 240 km (d) 150 dam

3. Express the following seale in the form of ratio :

(a) 1 cm = 5 m (b) 1 cm = $\frac{1}{2}$km (c) 1 cm = 1 km

(d) 1 cm = 80 m.

4. The scale of a diagram is 1 : 2000000. What is the actual distance represented by 2 cm in the diagram ?

5. The scale of a figure is 1 : 100000. The length of a road is 3·5 km. By what length can it be represented in the figure ?

6. The length and breadth of a sports arena are 120 m and 80 m respectively. Draw its diagram by taking a proper scale.

7. The length of a stream is 11·5 km and it is represented by 5 cm in a diagram. What is its scale?

Model diagram drawn to scale

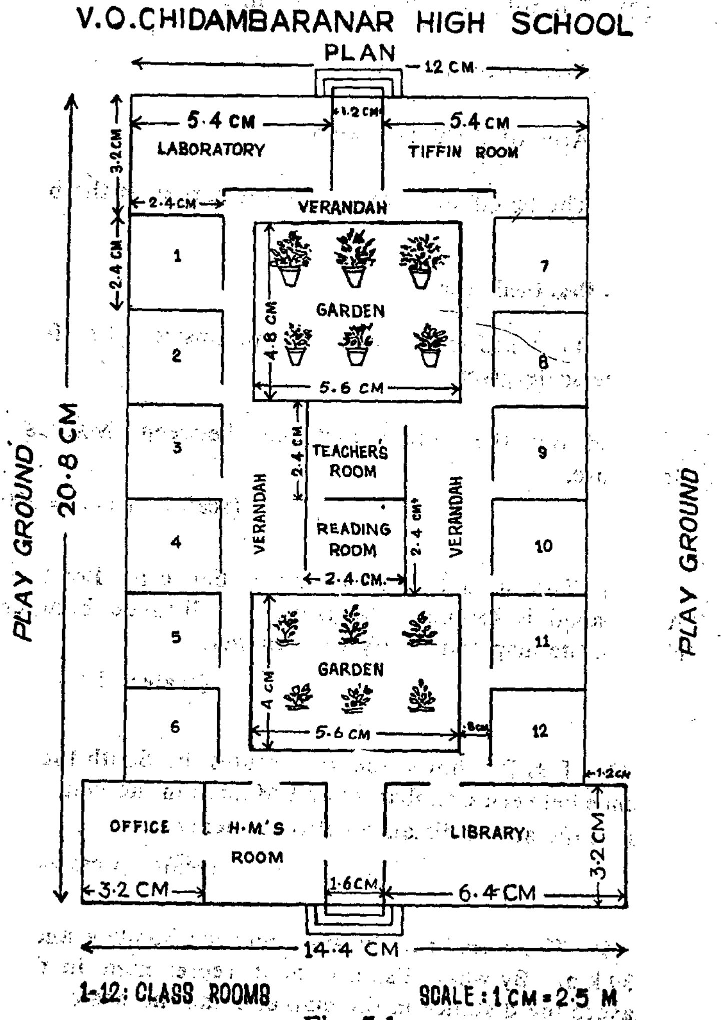


Fig. 7-1.

Scale — Practical Work (1)

The plan of a school is given in Fig. 7-1. Find the actual dimensions of the following :

(1) The front breadth of the school building

(2) The total length of the building

(3) Area of the library

(4) Area of each class room

(5) The length and breadth of the garden at the back

Scale — Practical Work (2)

(1) Look at Fig. 7-2 and try to answer the following from the scale given :

Find out the actual distance between Madras and Bangalore.

(scale : 1 cm = 300 km)

(2) The actual air distance between Bombay and Ahmedabad is 480 km. Measure the distance between the cities in the map and verify your answer.

(scale : 1 : 30 000 000)

(3) Fig. 7-3 shows the rail routes in South India. The distance between Coimbatore and Madras in the map is 1·6 cm. What is the actual distance between them ?

(scale : 1 cm = 310 km)

(4) The actual rail distance between Bombay and Nagpur is 830 km. By what distance is it represented in the map ? Measure the distance in the map and find the scale.

INDIA — AIR ROUTES

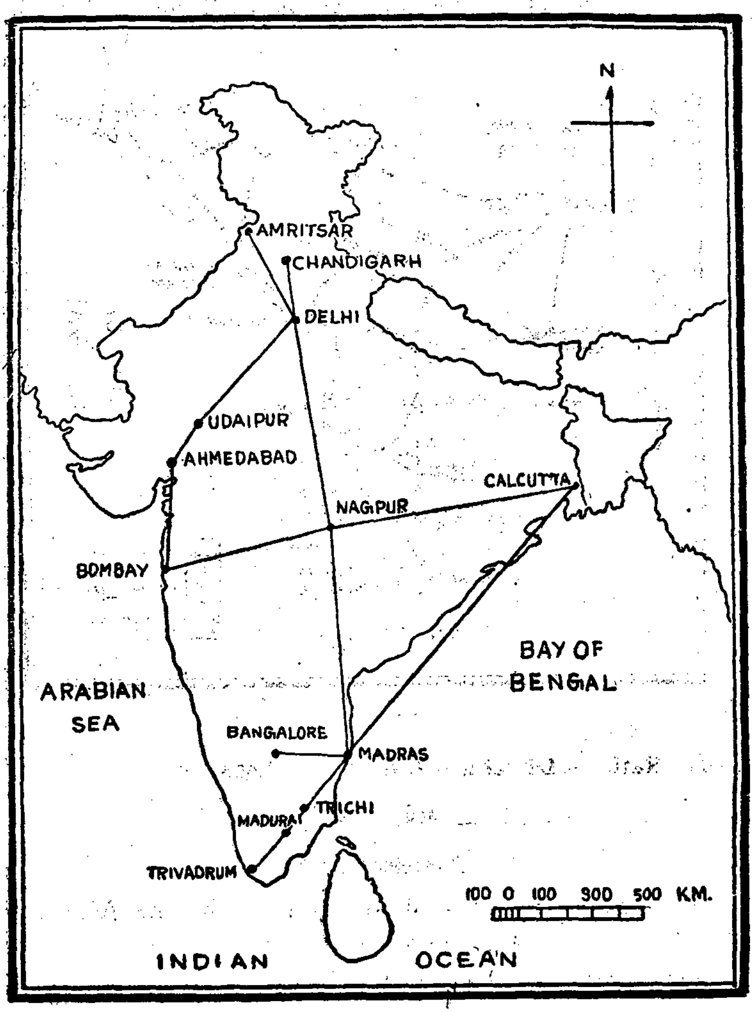


Fig. 7-2.

SOUTH INDIA — IMPORTANT RAIL ROUTES

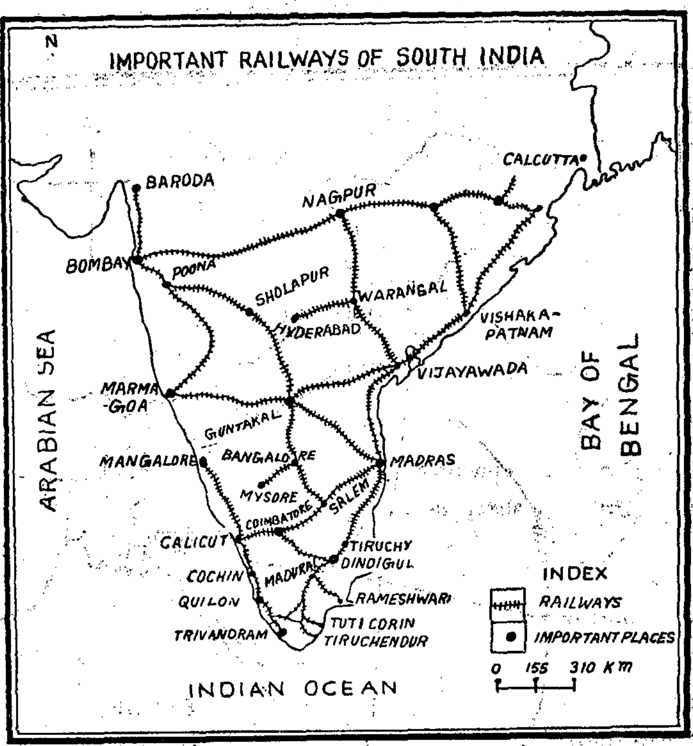


Fig. 7-3.

2—1. Ratio — Direct and Inverse Variations

PRE - TEST

Exercise 2—1

1. Determine the kind of variations in the following (mention if it is direct or inverse):

(a) Quantity and cost of the things.

(b) Height of poles and their shadows at a particular time

(c) The speed of the vehicle and the time taken for covering a certain distance

(d) The number of members in a family and the monthly expenditure

(e) The number of pages and the total number of lines in a page

(f) Age of a person and his income

(g) The cost of a book and the number of pages in the book

(h) Number of working hours of a day and the time taken to complete a work

2. If the cost of 18 kg of sugar is Rs. 57-60, what is the cost of 27 kg of sugar ?

3. The total daily wages of 15 men is Rs. 135 and the total daily wages of 20 women is Rs. 120. What is the total daily wages of 45 men and 30 women ?

4. A novel can be printed in 400 pages with 36 lines per page. How many lines will there be in a page if the same is reprinted in 360 pages ?

5. 32 men can repair a road in 18 days. In how many days can the same be repaired if 8 of them did not turn up for work ?

6. The length of a shadow of a tree of height 15 m is 18 m. What would be the length of the shadow of a tree of height 10 m at the same time ?

7. 12 carpenters did a work in 9 hours. In how many hours will the same work be done by 8 carpenters ?

8. A certain distance can be covered in 12 hours at a speed of 30 km/hour. If the speed is increased to 40 km/hour, what would be the time taken to cover the same distance ?

2–2. Compound Variation

Example 1 :

The eost of a plot of length 8 m and breadth 5 m is Rs. 1200. What is the cost of the plot of dimensions 12 m × 8 m?

Let us tabulate the above particulars :

Length of the plot	Breadth	Cost
8 m	5 m	Rs. 1200
12 m	8 m	?

As the length increases from 8 m to 12 m, its cost also increases by $\frac{12}{8}$ (direct variation).

In the same way, as the breadth increases from 5 m to 8 m, its cost increases $\frac{8}{5}$ times (direct variation).

Total increase in price $= \frac{12}{8} \times \frac{8}{5}$ times.

$\therefore$ The cost of the 2nd plot $= \text{Rs. } \frac{12}{8} \times \frac{8}{5} \times 1200$

$= \text{Rs. } 2880.$

Example 2 :

The expenses for a family of 10 members for 15 days is Rs. 900. What would be the expenses for a family of 6 members for 20 days at the same rate ?

Members	Days	Expenses
10	15	900
6	20	?

As the number of members is reduced from 10 to 6, the expenses will also be reduced to $\frac{6}{10}$ times (direct variation).

As the number of days is increased from 15 to 20, the expenses would increase $\frac{20}{15}$ times (direct variation).

∴ Increase in total expenses $= \frac{6}{10} \times \frac{20}{15}$ times

∴ The total expenses for the second family

$$= \text{Rs. } \frac{6}{10} \times \frac{20}{15} \times 900$$

$$= \text{Rs. } 720.$$

Example 3 :

17 men construct a house in 60 days working 9 hrs. a day. In how many days can 18 men working 10 hrs. a day construct the same building ?

Men	Hours of work/day	Days
17	9	60
18	10	?

∴ More men can finish the work in less number of days (inverse variation).

Since the number of men at work is increased from 17 to 18, the number of days taken to complete the work will be reduced to $\frac{17}{18}$ times.

The more the working hours per day, the less the number of days for the completion of the work.

Since the number of working hours per day is increased from 9 to 10, the number of days taken to complete the work will be reduced by $\frac{9}{10}$ times.

∴ Number of days taken by 18 men working 10 hrs. a day to complete the building

$$= \frac{17}{18} \times \frac{9}{10} \times 60$$

$$= 51 \text{ days.}$$

Exercise 2—2

Oral :

1. The cost of 12 electric bulbs is Rs 42. What is the cost of 18 bulbs at the same rate ?

2. The price of 10 note books is Rs. 15. What is the price of 25 note books at the same rate ?

3. 16 men can complete a piece of work in 20 days. In how many days can 8 men do the same work ?

4. A person takes 5 hrs. to reach Pollachi from Coimbatore travelling at 8 km/hour. How long will be take to return to Coimbatore if he travels at a speed of 10 km/hour ?

5. At a certain rate of interest, the interest for Rs. 500 is Rs. 120 for 2 years. Find the interest for Rs. 400 at the same rate and for the same period.

6. I could buy 24 fruits at the rate of 20 paise for the money I have. If the price of a fruit is 15 paise how many fruits could I buy for the same amount ?

Written :

7. The expense for printing 500 copies of a book each containing 100 pages is Rs. 750. What is the cost for printing 400 copies of the same book each containing 150 pages ?

8. 49 people earn Rs. 9800 in 25 days. How much will 35 people earn in 30 days ?

9. The electricity charges for 6 bulbs burning 8 hours a day is Rs. 24 per month. What will be the monthly electricity charge, when only 5 bulbs burn 6 hours a day ?

10. In a hostel there was enough food for 140 students for 30 days each consuming 400 grams a day. 40 of them left the hostel on vacation. For how many more days will the same quantity of food be sufficient for the remaining persons if each of them consumes 420 grams a day ?

11. A contractor engaged 40 men to repair a road of length 4 km in 10 days. In how many days will half the number of previous men repair a road 3 times the length of the original road ?

2—3. Time and Distance

The distance covered by a person depends on his speed and travelling time. Hence, time, distance and speed are inter related. Let us find the relations among them.

The distance covered in 1 hour is called speed and is represented as km / hour.

$$\text{distance} = \text{speed} \times \text{time}$$

$$d = s \times t$$

$$\text{speed } (s) = \frac{\text{distance}}{\text{time}} = \frac{d}{t}$$

$$\text{time } (t) = \frac{\text{distance}}{\text{speed}} = \frac{d}{s}$$

If any two of the three are known, the third one can be easily found out.

The following table shows the relationship between time and distance

s = speed = 30 km / hour.

t (hours)	6	8	2	3	10
d (km)	180	240	60	90	300

Calculate the ratios of the time and distance.

The following table gives the different times taken to cover 240 km at varying speeds :

Distance $d = 240$ km.

s	30 km/hr	60	20	15	40	24
t	8	4	12	16	6	10

Calculate the ratios between speed and journey time. From the above table we learn :

When distance travelled remains the same —

1. (a) As the speed increases, journey time decreases proportionately

   (b) As the speed decreases, journey time increases proportionately

2. The speed varies inversely as time.

Example :

Express the speed of 1 km per hour in terms of metres per second.

1 hour = 60 × 60 seconds

1 km = 1000 metres

∴ Distance covered in 60 × 60 seconds = 1000 metres

∴ Distance covered in 1 second $= \frac{1000}{60 \times 60}$ m

∴ Speed $= \frac{5}{18}$ m/sec.

1 km / hour $= \frac{5}{18}$ metre / sec.

Distance covered in 1 minute $= \frac{5}{18} \times 60$ m

$= \frac{50}{3}$ m.

$$1 \text{ km / hour} = \frac{50}{3} \text{ metre / minute}$$

Express 1 metre per minute as so many kilo metres per hour.

Distance covered in 1 second = 1 m.

Distance covered in 3600 seconds or 1 hour

$$= 1 \times 3600 \text{ metres}$$

$$= \frac{3600}{1000} \text{ km}$$

$\therefore$ Speed = 3·6 km/hour

$$1 \text{ metre / second} = 3{\cdot}6 \text{ km / hour}$$

Example 1 :

A person starts from P at 11.30 a. m. and travels at 36 km / hour. On the way he takes rest for 30 minutes and then again sets off at the same speed. Then he reaches Q at 3.30 p. m. What is the distance from P to Q ?

Total time taken for the journey

= from 11.30 a.m. to 3.30 p.m.

= 4 hrs.

Time taken for rest = 30 minutes or $\frac{1}{2}$ hr.

$\therefore$ Time taken for travel = $\left(4 - \frac{1}{2}\right) = 3\frac{1}{2}$ hrs.

Speed = 36 km/hour

$\therefore$ Distance covered in $3\frac{1}{2}$ hrs.

or

Distance between P and Q

$$= 36 \times 3\frac{1}{2} = 126 \text{ km.}$$

Example 2 :

The distance between two places is 120 km. A travels the distance at 30 km / hour. B takes 1 hr. more than A to cover the same distance. Find the speed of B ?

Total distanee = 120 km.

Speed of A = 30 km/hr.

Time taken by A $= \frac{d}{s} = \frac{120}{30} = 4$ hrs.

Time taken by B $= 4 + 1 = 5$ hrs.

Distance covered by B in 5 hours = 120 km

$\therefore$ Speed of B $= \frac{d}{s} = \frac{120}{5} = 24$ km/hr.

Example 3 :

A person travels a distance of 175 km. After travelling 50 km at a speed of 20 km/hr, he completes his journey at a speed of 25 km/hr. Find the time taken by him for the journey.

Total distance = 175 km

Time taken to cover 50 km at 20 km/hour

$$\frac{d}{s} = \frac{50}{20} = 2\frac{1}{2} \text{ hours}$$

Remaining distance $= 175 - 50 = 125$ km

Time taken to cover 125 km at 25 km/hour

$$= \frac{125}{25} = 5 \text{ hours}$$

Total time taken $= 2\frac{1}{2} + 5$ hours $= 7\frac{1}{2}$ hours

Exercise 2—3

Oral :

1. Convert the following speeds into metres per second :

(a) 18 km/hour (b) 36 km/hour
(c) 45 km/hour (d) 72 km/hour

2. Convert the following into km/hour :

(a) 5 m/sec (b) 35 m/sec (c) 15 m/sec
(d) 25 m/sec

3. What is the distance covered in $2\frac{1}{2}$ hours by a person travelling at 25 km/hour ?

4. Find the time taken to travel 100 km at a speed of 20 km/hour.

5. What is the speed if a person travels 50 km in 2 hours ?

6. A man took 2 hours 20 minutes to reach from one place to another at a speed of 60 km. by car. What is the distance between these places ?

Written :

7. What is the speed if a person reaches a place 80 km away in 2 hours 30 minutes ?

8. An aeroplane travels at a speed of 180 km/hour. Find its speed in metre / second and in km/minute.

9. The distance between P and Q is 200 km. A starts from P at 4 a.m. and travels at a speed of 40 km/hour and reaches Q. He relaxes for 3 hours at Q. When will he return to P, if he travels back at a speed of 50 km / hour ?

10. The distance from the foot of a hill to its top is 90 km. A person starts from the foot of the hill at 6 a.m. and climbs up to the top at a speed of 15 km / hour. Then he takes rest at the top for 2 hours. When will he reach the foot of the hill climbing down at a speed of 20 km/hour ?

11. A passenger starts from Poongulam at 12.30 p.m. and travels at a speed of 40 km/hour. He reaches Thamaraikulam at 3 p.m. After resting there for 30 minutes he reaches Solaiyoor at 5 p.m., travelling at a speed of 60 km/hour. What is the distance between Poongulam and Salaiyoor ?

3 — 1. Simple Interest

Let us recall how we calculate the interest. The interest depends on the principal and the time taken to return the principal. For example, the interest of Rs. 8, charged for one year is noted as 8% interest. Banks and other organizations pay interest at a certain percentage.

In rural areas people give the rates of interest as 1 interest, $\frac{3}{4}$ interest, $\frac{1}{2}$ interest and $\frac{1}{4}$ interest. 1 interest means for every Re. 1 the interest is 1 paisa per month. $\frac{1}{2}$ interest denotes, for every Rs. 100 the interest will be $\frac{1}{2}$ Rupee per month.

Formula :

Let P denote the principal, n denote the number of years and i denote the rate of interest.

$$\therefore \quad \text{Interest } I = Pni$$

$$\text{Amount } A = P + Pni$$

To find the interest :

Example 1 :

Find the interest on Rs. 500 for 3 years if the rate of interest is 12% per annum.

$P = 500$

$n = 3$

$i = \frac{12}{100} = 0{\cdot}12$

$I = 500 \times 3 \times 0{\cdot}12 = 180$

Interest is Rs. 180

Example 2 :

Find the interest and the amount on Rs. 2400 for 2 years 6 months if the rate of interest is 8% p. a.

$$I = Pni$$

$$P = 2400; \quad n = 2\frac{1}{2}; \quad i = \frac{8}{100} = 0{\cdot}08$$

$$I = 2400 \times 2\frac{1}{2} \times 0{\cdot}08$$

$$= 2400 \times \frac{5}{2} \times 0{\cdot}08 = 480$$

Interest = Rs. 480

Amount $A = P + Pni$

= Rs. 2400 + Rs. 480 = Rs. 2880.

To find the principal :

Example :

A sum of Rs. 360 is realised as interest on a principal for 4 years, when the rate of interest is 15% p. a. Find the principal.

Let the principal be Rs. 100

Interest for 4 years at the rate of 15% p. a.

$$= \text{Rs. } 100 \times 4 \times \frac{15}{100} = \text{Rs. } 60.$$

If Rs. 60 is realised as interest, the principal is Rs. 100

∴ For Rs. 360 interest, the principal is

$$= \text{Rs. } 360 \times \frac{100}{60} = \text{Rs. } 600$$

Formula method :

Since $Pni = I$

$$P = \frac{I}{ni}$$

$$= \frac{360}{4 \times \frac{15}{100}}$$

$$= \frac{360 \times 100}{4 \times 15} = 600.$$

Principal is Rs. 600.

To find 'n' :

Example :

In how many years will Rs. 1200 yield an interest of Rs. 600 at 10% p. a. ?

1 year interest for Rs. 1200 at 10% p. a. = Rs. 120

For an interest of Rs. 120, the period = 1 year

For an interest of Rs. 600 the period

$$= 600 \times \frac{1}{120} = 5$$

No. of years is 5.

Formula method :

Since $Pni = I$

$$n = \frac{I}{Pi}$$

$$= \frac{600}{1200 \times \frac{10}{100}}$$

$$= \frac{600 \times 100}{1200 \times 10} = 5$$

No. of years is 5.

To find the rate of interest :

Example :

A sum of Rs. 800 yields Rs. 288 as 5% interest in 4 years. Calculate the rate of interest.

For Rs. 800, the interest for 4 years = Rs. 288

For Rs. 800, the interest for 1 year $= \text{Rs. } \frac{288}{4} = \text{Rs. } 72$

∴ For Rs. 100, interest for 1 year

$$= \text{Rs. } \frac{72 \times 100}{800} = \text{Rs. } 9$$

Rate of interest is 9 %

Formula method :

$$i = \frac{I}{Pn}$$

$$= \frac{288}{800 \times 4}$$

$$= \frac{9}{100}$$

Rate of interest is 9%

Exercise 3—1

Oral :

1. Find the interest on Rs. 600 for 2 years when the rate of interest is 10% p. a.

2. Find the interest on Rs. 1000 for 2 years when the rate of interest is $8\frac{1}{2}\%$ p. a.

3. Find the interest on Rs. 500 for 6 months when the rate of interest is 12% p. a.

4. Find the interest for Rs. 50 (1) for 6 months (2) for 3 months, when the rate of interest is 10% p. a.

5. Find the interest on Rs. 1200 for 3 years at the rate of 10% p. a.

6. Calculate the interest for 1 Rupee per month when the rate of interest is

(a) 6% (b) 12% (c) 9% (d) 18%

7. Find the rate of interest per annum, if

(a) for Re. 1 per month the interest is 1 paisa.

(b) for Re. 1 per month the interest is 2 paise.

8. Find the interest on Rs. 600 for 2 years at the rate of $6\frac{1}{4}\%$ p. a.

9. Find the interest on Rs. 900 for 10 years at the rate of 6% p. a.

10. Find the rate of interest per annum, if

(a) for Rs. 100 the quarterly interest is Rs. 3.

(b) for Rs. 100 the interest per month is Rs. 1-50.

(c) for Rs. 50 the interest for 3 years is Rs. 18·00.

11. A certain principal is doubled at some rate of simple interest in 20 years. Find the rate of interest.

12. Calculate the amount for 1 year for a sum of Rs. 500 when the interest is Re. 1 for Rs. 100 per mensem.

Written :

13. Find the interest :

	Principal	Period	Rate of interest
(a)	Rs. 2400	3 years	12%
(b)	Rs. 750	2 years 8 months	10%
(c)	Rs. 1200	3 years 4 months	8%
(d)	Rs. 300	3 years 6 months	10%
(e)	Rs. 1650	2 years	6%

14. A person borrows Rs. 1000 for 2 years at the rate of 1 paisa for Re. 1 per mensem. An equal sum is borrowed by another person for the same period but at a rate of 14% p.a. Who pays more interest and how much ?

15. Find the principal if Rs. 54 is got as interest in 2 years at the rate of 9% p. a.

16. What will be the principal if it gets an interest of Rs. 360 in 3 years at the rate of 6% p.a.?

17. Calculate the rate of interest if Rs. 400 yields Rs. 50 in $2\frac{1}{2}$ years.

18. Calculate the rate of interest

(a) if Rs. 500 yields Rs. 80 in 2 years

(b) if Rs. 200 doubles itself in 10 years at a certain rate of interest.

19. In what period will a certain principal double itself at 10% p. a. ?

20. A person borrowed Rs. 10,000 at 9% p. a. Of this he immediately lent Rs. 4000 to *B* at 12% p. a. and the balance at 15% rate of interest p. a. to *C*. After three years he collected the amounts due to him and repaid his loan. How much profit did he make in this transaction ?

21. A person will get Rs. 82.50 if he invests Rs. 50 in 12 year National Saving Bonds. Find the rate of interest.

22. In how many years will a principal Rs. 500 fetch an interest of Rs. 100 at 10% p. a. ?

3—2. Taxes

Taxes are sources of money for government and local bodies. Taxes are of two kinds.

(1) Direct tax (2) Indirect tax

Direct tax is collected directly from a person concerned. Income tax and House tax are examples of direct taxes. In indirect taxes, somebody collects it and pays to government. For example, a shopkeeper collects sales tax on articles at the time of sale from the buyer and pays the same to the government. Sales tax and Entertainment tax are examples of indirect taxes.

Example 1 :

A steel furniture dealer sold two almirahs at Rs. 900 each and 8 folding chairs at Rs. 55 each. Find how much tax does the dealer pay if the sales tax for an almirah is 15% and for the chair 10%.

Cost of 2 almirahs $= \text{Rs. } 900 \times 2 = \text{Rs. } 1800$

Sales tax at 15% $= \text{Rs. } \dfrac{1800 \times 15}{100} = \text{Rs. } 270$

Cost of 8 Chairs $= \text{Rs. } 55 \times 8 = \text{Rs. } 440$

Sales tax at 10% $= \text{Rs. } \dfrac{440 \times 10}{100} = \text{Rs. } 44$

Total sales tax $= \text{Rs. } 270 + \text{Rs. } 44 = \text{Rs. } 314.$

Example 2 :

Find the amount of tax to be paid by a merchant if the tax is collected at the rate of 6% on selling price. The merchant sold two electric fans at Rs. 450 each and 2 electric tube lights at Rs. 50 each.

Selling price of 2 electric fans = Rs. 900

Selling price of 2 tube lights = Rs. 100

Total selling price = Rs. 1000

Sales tax at the rate of 6% $= \text{Rs. } \dfrac{1000 \times 6}{100}$

$= \text{Rs. } 60$

Exercise 3—2

1. The sales tax on leather products is 8%. If a person buys 6 hand bags at Rs. 30 each, 100 cash bags at Rs 12 each and 50 waist belts at Rs. 5 each, find the amount of tax he has to pay.

2. An invoice of M/s. Girinath Drug Shop is given below. Calculate and fill up the blanks suitably.

Particulars	Rate	Quantity	Amount
Horlicks bottle (Small size)	Rs. 12.25 each	10 bottles	....
Horlicks bottle (Giant size)	Rs. 21.50 each	10 bottles	....

Sales Tax 4% ....

Total Rs. ....

3. The following articles are purchased at Shanthi Ready made Mart. Sales tax is collected at 2% on all items. Find the total tax.

(1) 5 baby suits at Rs. 40 each

(2) 6 frocks at Rs. 25 each

(3) 10 banians at Rs. 4·50 each

(4) 4 pillow covers at Rs. 2·50 each

4. An oil mill manufactures 150 quintals in each of the following kinds of oil — groundnut oil, castor oil and coconut oil. The cost of groundnut oil is Rs. 900 per quintal; the cost of castor oil is Rs. 1250 per quintal; the cost of coconut oil is Rs. 1650 per quintal. Find how much sales tax he has to pay to the government at the rate of 4%.

5. The cost price of a wet grinder is Rs. 1500. If the price of the same is Rs. 1680 inclusive of sales tax, what is the rate of sales tax ?

6. A shop keeper collects Rs. 15·40 by way of sales tax on the plastic articles he has sold on a particular day. If the sales tax is collected at the rate of 4%, find the total sale on that day.

7. The total value of houses in a village for which the tax is collected is Rs. 5,00,000. Determine the rate of tax for every Rs 100, if the total house tax collected is Rs. 20,000.

3 — 3. Exchange

Every country has got its own system of currency. For example the Rupee is our currency. The following are the various currencies being used in several countries :

Country	Currency
Britain	Pound
U. S. A.	Dollar
Japan	Yen
West Germany	Mark
Russia	Rouble

There is a relationship between any two currencies by which we can exchange one currency for another. This is called 'Exchange'. The rates of exchange will be changing from time to time.

For example, '1 dollar = Rs. 7·90' is the exchange rate for U. S. Dollars. This means that Rs. 7.90 has to be paid in Indian money to purchase an article worth 1 American dollar.

Example 1 :

Electrical instruments are purchased from Japan for the value of 50,000 yen. Express its value in Indian money (100 yen = Rs. 3·30).

$$100 \text{ yen} = \text{Rs. } 3{\cdot}30$$

$$\therefore \quad 50000 \text{ yen} = \text{Rs. } \frac{3{\cdot}30}{100} \times 50000 = \text{Rs. } 1650$$

Example 2 :

Handloom garments worth Rs 1,58,000 are exported to United States from India. Find the value of the same in U. S. dollars. (Rs. 7·90 = 1 dollar)

$$\text{Rs. } 7{\cdot}90 = 1 \text{ dollar}$$

$$\text{Rs. } 158000 = \frac{1}{7{\cdot}90} \times 158000 = 20000 \text{ dollars.}$$

Example 3 :

Find how much should be paid in Indian currency to buy a German camera at 180 Marks each. (1 Mark = Rs. 4·50)

$$1 \text{ Mark} = \text{Rs. } 4{\cdot}50$$

$$\therefore \quad 180 \text{ Marks} = \text{Rs. } 4{\cdot}50 \times 180$$

$$= \text{Rs. } 810$$

∴ The cost of a German camera in Indian currency is Rs. 810.

Exercise 3—3

1. Food grains worth Rs. 6,32,00,000 are imported from U.S.A. Express the value in dollars. (1 dollar = Rs. 7·90)

2. A machine is imported from West Germany and its cost in Germany is 24,000 Marks. Find its price in our currency. (1 Mark = Rs. 4·50)

3. A tape recorder costs 44,000 yens in Japan. How much should we pay in Rupees for the same. (100 yen = Rs. 3·30)

4. Calculate the amount in Italian currency if jute worth Rs. 17,500 is exported to Italy. (1 Re. = 108 liras)

5. Machines worth Rs. 3,60,000 were sent to Australia from India. Find their price in Australian dollars.

(1 Australian dollar = Rs. 9)

3 — 4. House Rent

A tenant pays a certain amount every month to the house owner for its use. The money paid every month is called rent. The land owner is offered a certain amount and a part of the yield for the cultivation of his land. The rest of the yield and other income derived from the land are retained by the person who cultivates it and he is called the lessee. The amount offered is the lease amount.

Normally a government servant who lives in a building owned by the government pays his house rent allowance plus a portion of his salary in a certain ratio every month as house rent.

Example :

A person pays 12% of his monthly salary as house rent. If his monthly salary is Rs. 540, find how much he pays as house rent ?

$$\text{Monthly rent} = \text{Rs. } 540 \times \frac{12}{100}$$

$$= \text{Rs. } \frac{6480}{100} = \text{Rs. } 64{\cdot}80$$

Exercise 3—4

1. The monthly income of a government servant is Rs. 640. Find how much monthly rent would he pay at the rate of 10%. Find his net ineome.

2. The monthly income of a person is Rs. 450. Find at what rate he pays the house rent if he pays Rs. 90 monthly.

3. A person owns 5 hectares of wet lands and 12 hectares of dry lands. He is paid at the rate of Rs. 2500 per hectare for wet lands and Rs. 450 per hectare for dry lands. What is the lease amount received by him ?

3—5. Insurance

We know a family will be at a great loss if the head of the family dies in harness . If godowns or shops are burnt down as a result of fire accidents, the owner incurs a heavy loss. Likewise due to natural calamities people involved in them lose heavily. To get relief from such losses, people insure their lives or goods with an Insurance Company and pay a certain amount (called premium) to the insurance company. Whenever they incur a loss, the insurance company makes good the loss.

Losses due to death, fire and accident will not usually occur often. Hence the insurance company compensates the loss from the premium amount collected.

Insurance is of various kinds — Life Insurance, Fire Insurance, Marine Insurance, Motor Vehicle Insurance and Crop Insurance. The Life Insurance Corporation introduces various new schemes every year.

Life Insurance Corporation — Life Insurance

In 1956 life insurance in India was nationalised. Now the same is functioning under the name, "Life Insurance Corporation".

People get several benefits from life insurance schemes. They are :

(a) It is a form of compulsory small savings

(b) It helps people quite a lot during their old age

(c) When the head of a family dies in harness, the insured amount is of great help to his dependents

(d) Income tax exemption is allowed for the premium amount

(e) Loan can be raised on the insurance policy by a policy holder

(f) The premium amount collected from the people is utilised by the corporation for various developmental activities, such as the five year plans, etc.

A person who has insured his life pays the premium amount at regular instalments for a certain period, say for 15 years, 20 years, 25 years, etc. If an insured person dies before the end of the period upto which he has to pay the premium, the insurance company pays the full insured amount. The amount of premium to be paid depends on the age of a person and the number of years as mentioned in the table.

LIFE INSURANCE POLICY [With Endowment]

PREMIUM FOR EVERY RUPEES 1000 PER YEAR

Age of policy holder	No. of years the premium is to be paid.											
	5		10		15		20		25		30	
	Rs.	P.	Rs.	P.	Rs.	P.	Rs.	P.	Rs	P.	Rs.	P.
16	219	44	108	00	70	81	52	19	41	12	33	87
17	219	44	108	00	70	81	52	19	41	12	33	94
18	219	44	108	00	70	81	52	19	41	12	34	00
19	219	44	108	00	70	81	52	19	41	19	34	06
20	219	44	108	00	70	81	52	25	41	25	34	12
21	219	44	108	00	70	81	52	31	41	31	34	25
22	219	50	108	00	70	87	52	37	41	37	34	37
23	219	50	108	00	70	87	52	44	41	50	34	50
24	219	50	108	00	70	94	52	50	41	62	34	62
25	219	50	108	06	71	00	52	56	41	75	34	75
26	219	56	108	06	71	00	52	69	41	87	34	94
27	219	56	108	12	71	06	52	81	42	00	35	12
28	219	56	108	19	71	12	52	94	42	19	35	37
29	219	62	108	25	71	25	53	06	42	37	35	62
30	219	62	108	31	71	37	53	19	42	56	35	94
31	219	69	108	37	71	50	53	37	42	81	36	31
32	219	75	108	44	71	62	53	56	43	12	36	75
33	219	81	108	56	71	81	53	81	43	44	37	19
34	219	87	108	69	72	00	54	06	43	81	37	69
35	219	94	108	87	72	25	54	37	44	25	38	19

General Insurance Corporation deals with insurances other than Life Insurance. The premium amount will be a certain percentage of the value of the goods so insured in general insurance.

Example 1 :

A building is insured for Rs. 1,00,000 and the rate of premium is 35 paise per Rs. 1000 per year. What will be the amount of half yearly premium ?

Value of the goods insured = Rs. 1,00,000

Rate of premium = 35 paise per Rs. 1000

$\therefore$ Yearly premium = Rs. $1,00,000 \times \frac{35}{1000}$

= Rs. 35

$\therefore$ Half yearly premium = Rs. $35 \div 2$

= Rs. 17·50

Example 2 :

Find the yearly premium for an insurace policy of value Rs. 50,000 at the rate of 3% per year.

Value of the policy = Rs. 50,000

Rate of premium = 3%

$\therefore$ Yearly premium = Rs. $\frac{3}{100} \times 50,000$

= Rs. 1500.

Example 3 :

Refer to the insurance table and find the yearly premium for a 25 years old person who wants to insure his life for 30 years. Calculate also the yearly premium for him if he would like to insure his life at the age of 30 for a period of 25 years.

Yearly premium for a 25 years old person whose insurance period is 30 years for Rs. 1,000 = Rs. 34·75

$\therefore$ Yearly premium for Rs. 10,000 = Rs. 347·50

Yearly premium for a 30 years old person for a period of 25 years for Rs. 10,000 = Rs. 42·56 × 10

= Rs. 425·60

Exercise 3—5

1. An educational institution insured its buildings for Rs. 2,00,000 at the rate of premium 30 paise per Rs. 1000 per year. Calculate the yearly premium.

2. Calculate the rate of premium per year when the owner of a building insured for Rs. 75,000 pays Rs. 22·50 as yearly premium.

3. A person insures five of his cows for Rs. 2,000 each. If the rate of premium is 3·25%, find the premium.

4. Calculate from the table the premium to be paid every year in each of the following cases :

(a) 30 years old person takes a policy for Rs. 4,000 for 20 years

(b) A 27 years old person takes a policy for Rs. 8,000 for (i) 20 years (ii) 30 years

(c) A 25 years old person insures his life for Rs. 10,000 for 30 years. Find the quarterly premium.

(d) A municipal servant takes a policy for Rs. 20,000 for 25 years at the age of 30 and requests the municipality to deduct the premium amount from his salary. Find how much will be deducted from his salary every month.

5. A person insured his goods for Rs. 36,000 at the rate of $1\frac{3}{4}$ % on the value. Find the premium.

6. Calculate the rate of percentage of premium for the goods of value Rs. 48,000 if the yearly premium is Rs. 600.

3—6. Commission

For a person who helps in buying or selling a house or a land, a certain amount is usually paid as commission. This amount is based on the sale amount. One who does this job is called an **Agent** and the money he gets in return for his service is called the **Commission.**

Generally the agent gets his commission from both the buyer and the seller. Commission is calculated either for every Rs. 100 or for a rupee.

Exercise 3—6

1. Find the commission for Rs. 5,500 at the rate of 5%.

2. Find the rate of commission for Re. 1 at 10%.

3. An agent sold goods for Rs. 15,000. If he gets a commission on the sales at 2%, how much does he get ?

4. An agent sold a house for Rs. 50,000. How much does he get if he receives commission at 2% from both the buyer and the seller ?

3—7. Percentages — Application

Example 1 :

From the following table you can find the price at present and also the percentage of increase in the prices between now and 4 years back. Find the price of each of the article 4 years back.

Article	Increase in price	Price at present
Rice	50%	Rs. 2·70 / kg
Milk	25%	Rs. 2.30 / *l*
Sugar	$33\frac{1}{3}$%	Rs. 6·00 / kg
Gingily oil	40%	Rs. 14·00 / kg

Let the price of rice per kg before 4 years be Rs. 100.

$$\text{Increase in price} = 50\%$$

$$\therefore \quad \text{Rate of increase in price} = \frac{150}{100}$$

$\therefore$ Price at present $= \text{Rs. } 2{\cdot}70$

Price 4 years back $= \text{Rs. } \frac{100}{150} \times 2{\cdot}70$

$= \text{Rs. } 1{\cdot}80.$

Similarly the price of the other three commodities can be found.

Example 2 :

The population of a village at the beginning of 1977 was 20,000 and it decreased by 10% by the end of 1977. Again it increased during 1978 by 15%. Find the population at the beginning of 1979 ?

Population at the beginning of 1977 $= 20{,}000$

Decrease in population during the year $= 10\%$

$\therefore$ Population at the end of 1977

$$= 20{,}000 \times \frac{90}{100} = 18{,}000$$

Increase in population during 1978 $= 15\%$

$\therefore$ Population at the end of 1978 or at the beginning

$$\text{of } 1979 = 18{,}000 \times \frac{15}{100} = 20{,}700.$$

Exercise 3—7

Oral :

1. Find the percentage of savings of a person if he spends 40% of his income ?

2. Express as percentages :

$\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{20}, \frac{1}{3}, \frac{2}{3}, \frac{2}{5}, \frac{1}{8}, \frac{9}{20}, \frac{19}{20}.$

3. (a) Express as fractions :

$5\%, 10\%, 25\%, 20\%, 12\frac{1}{2}\%, 6\frac{1}{4}\%, 99\%.$

(b) Convert into percentages :

0.07, 0.24, 0.12, 0.375, 0.99, 0.625.

4. **Find 10% of Rs. 40.**

5. A student got 16 marks out of 80. Express it as a percentage.

6. Find:
   (a) 25% of a metre
   (b) 40% of 75 marks
   (c) $33\frac{1}{3}$% of a year
   (d) 25% of Rs. 400
   (e) 20% of a kg
   (f) 20% of Rs. 20
   (g) 15% of a crowd of 1500
   (h) 25% of 1 hour 40 minutes
   (i) 25% of 50% of 20 litres

7. Find 10% of 5 and also 5% of 10.

8. 20% of the trees in a garden are fruit trees. Find the number of fruit trees if the total number of trees is x.

Written:

9. There are 2,500 trees in a garden. 20% of them are coconut trees, 10% palm trees and 40% mango trees. Find the number of other kinds of trees.

10. The total number of voters in an election is 6,000. The ratio of males and females is 2 : 1 10% of the males and 5% of the females do not exercise their votes. Find how many males and females have voted ?

11. There are 420 students in VI Standard in a school. 15% of them have failed. Find the number of students who passed.

12. 1,20,000 students remitted the examination fees. 90% of them attended the examination. Among these 52% passed. Find how many students failed.

13. A motor car costs Rs. 50,000 now. Its value decreases by 10% every year. Find the value of the car after two years.

14. The population of a town in 1971 was 2,75,000. The population of the villages surrounding the town was 1,25,000. During the next census it was found that there was an increase of 15% in the population of the town and 20% decrease in the population of the villages. Find the population of the town and the villages separately.

15. The population of a village is 8,400. The birth rate and the death rate are 5% and 2% respectively and the percentage of the new entrants is 1%. What will be its population after one year ?

16. The old price, the current price and the percentage of increase or decrease of certain commodities are given below. Fill up the blanks suitably :

Commodities	Old price	Increase % or Decrease %	Current price	Increase or Decrease
1. Rice	Rs. 2/kg.	20% (+)	—	—
2. Jaggery	—	50% (+)	Rs. 3/kg.	—
3. Toordal	Rs. 5-50/kg.	—	Rs. 5/kg.	—
4. Tamarind	—	25% (+)	Rs. 7-50/kg.	—
5. Ghee	Rs. 22/kg.	—	—	Rs. 1-10 (−)

17. The price of a bicycle is Rs. 500 and it is increased by 20%. If it is sold at 10% discount what is its price ? What is the pecentage of net increase in price ?

3–8. Percentages — Profit and Loss

We have already studied about problems involving profit and loss in business.

Traders buy things and sell them. The price at which an article is bought is called the **cost price** (C. P.) of the article.

If the **selling price** (S. P.) of the article is more than the cost price, then the trader gets profit. If the S. P. is less than the C. P. then the trader incurs a loss.

The term "Percentage" is often used in business. In trade problems we normally come across the C. P., S. P., profit or loss. If any two of the three is known the third one could be found.

C. P. + Profit (gain)	= S. P.
C. P. − Loss	= S. P.

Profit or Loss percentage is calculated only on the C. P.

1. Finding the S. P. when the C. P. and Profit/Loss% is given :—

(a) If there is gain%

$$\text{S. P.} = \frac{100 + \text{gain}\ \%}{100} \times \text{C. P.}$$

(b) If there is loss%

$$\text{S. P.} = \frac{100 - \text{loss}\ \%}{100} \times \text{C. P.}$$

2. Finding the C. P. when the S. P. and Profit / Loss % is given:

(a) If the gain % is given,

$$\text{C. P.} = \frac{100}{100 + \text{profit}\ \%} \times \text{S. P.}$$

(b) If there is loss %

$$\text{C. P.} = \frac{100}{100 - \text{loss}\ \%} \times \text{S. P.}$$

To find the selling price:

Example 1:

A man bought a camera for Rs. 300 and sold it at a profit of 20%. Find the S. P.

Method I

C. P. = Rs. 300

Profit = 20%

If the C. P. is Rs. 100, then the S. P. = Rs. 100 + 20
= Rs. 120.

If the C. P. is Rs. 300 then the S. P. $= \text{Rs. } 300 \times \frac{120}{100}$
= Rs. 360

Method II

$$\text{S. P.} = \text{C. P.} \times \frac{(100 + \text{gain }\%)}{100}$$

$$= \text{Rs. } 300 \times \frac{120}{100} = \text{Rs. } 360.$$

To find the cost price :

Example 2 :

A dealer sold a motor cycle for Rs. 5,500 thereby making a profit of 10%. Find the C. P. of the motor cycle.

Method I

Profit = 10%

If the S. P. is Rs. 110, then the C. P. = Rs. 100

If the S. P. is Rs. 5500, then the C. P.

$$= \text{Rs. } \frac{100}{110} \times 5500$$

= Rs. 5000.

Method II

$$\text{C. P.} = \frac{100}{100 + \text{Profit}\%} \times \text{S. P.}$$

$$= \text{Rs. } \frac{100}{110} \times 5500 = \text{Rs. } 5000.$$

Exercise 3—8

Oral:

1. Find the profit % if an article is bought for Rs. 75 and sold for Rs. 90.

2. A trader bought a bicycle for Rs. 500 and sold it for Rs. 550. Find the percentage of profit.

3. Fill in the blanks suitably :

S. No.	Particulars	C. P. Rs.	S. P. Rs.	Profit / loss Rs.
1.	Radio	800	700	
2.	Chair	60	66	
3.	Table	200	226	
4.	Steel Almirah	800	900	
5.	Clock	150	180	
6.	Hand bag	30	27	

4. The percentage profit is given below. By what fraction should the C. P. be multiplied to get the S. P. ?

(a) 5% (b) 20% (c) 25% (d) 7% (e) $12\frac{1}{2}\%$

(f) $33\frac{1}{3}\%$

5. The percentage of loss is given below. To get the S. P. by what fraction should the C. P. be multiplied ?

(a) 5% (b) 20% (c) 30% (d) 12% (e) 15%
(f) 18%

6. The following are the percentage of profit. By which fraction should the S. P. be multiplied to find C. P. ?

(a) 11% (b) 30% (c) 4% (d) 18% (e) $5\frac{1}{2}\%$
(f) 7%

7. The following are the percentage of loss. By what fraction should the S. P. be multiplied to find the C. P. ?

(a) 9% (b) 10% (c) 25% (d) 20 (e) 12%
(f) 30%

Written :

8. A person sells a house plot for Rs. 3,600 at a profit of 20%. Find the C. P.

9. A trader looses 10% when he sells a garden for Rs. 3,600. Find the C. P. of the garden.

10. 10% profit is realised when a motor car is sold for Rs. 33,000. What should be the S. P. if he desires to get a profit of 15% ?

11. The C. P. and profit / loss % of some articles are given below. Find the S. P.

	C. P.	Profit / loss%
(a)	Rs. 540	10% loss
(b)	Rs. 9000	20% profit
(c)	Rs. 460	8% loss
(d)	Rs. 60·50	4% profit

12. The S. P. and the profit/loss % of some articles are given below. Find the C. P.

	S. P.	Profit / loss %
(a)	Rs. 850	15% loss
(b)	Rs. 69	8% loss
(c)	Rs. 7·25	16% gain
(d)	Rs. 1150	15% gain

13. A fruit merchant bought 5000 oranges at Rs. 25 per hundred. Then he divided all the oranges into three qualities. 300 oranges became rot. 1000 first quality oranges were sold at Rs. 50 per hundred. 2000 second quality oranges were sold at Rs. 40 per hundred and the remaining oranges were sold at Rs. 30 per hundred. Find the percentage of profit ?

14. A timber merchant bought teak in an auction for Rs. 18,000. Then he met the following expenses while making artices, such as chairs, tables etc.

Transport charges	Rs. 500
Sawing charges	Rs. 200
Carpentry charges	Rs. 2300
Other manufacturing charges	Rs. 1000

He sold 100 tables at Rs. 200 each and also 100 chairs at Rs. 86 each. What is his percentage of profit ?

15. A dealer sells two transistor radios at Rs. 500 each and gets a profit of 20% on one and 25% loss on the other. Find his net percentage of profit / loss in the transaction.

16. A person sells a table for Rs. 180 at a loss of 10%. What should be the S.P. if 10% profit is expected in the sale ?

17. A dealer sells pencils at Rs. 5 a dozen and gets a profit of 25%. What would be the percentage of profit if each pencil is sold at 50 paise ?

TEST PAPER 7 (I)

[Time 45 minutes

1. 5 cm length in a map represents an actual distance of 2 km. What is the scale ?

2. A map is drawn to a scale 1 cm = 100 km. What is the distance between two places in the map if they are 1500 km apart ?

3. A tap takes $3\frac{3}{4}$ hours to fill up a water tank of capacity 250 litres. What time would be taken by the tap to fill up another water tank of capacity 400 litres ?

4. 28 men can complete a piece of work in 20 days. How many more men are required if the same work is to be completed in 16 days ?

5. The distance between *A* and *B* is 120 km. A man starts from *A* at 8 A.M. and proceeds to *B*. He relaxes for 30 minutes on the way. Then he continues his journey at the same speed and reaches *B* at 12-30 P.M. What is his speed during the journey ?

6. A person invested Rs. 20,000 on 1—1—78 in fixed deposit in a bank which gave interest at the rate of 12% p. a. He withdrew the amount along with the interest on 1—1—80 and raised a loan of Rs. 2,600 for buying a house, since the money was not found sufficient. What was the total cost of the house ?

7. The value of a vacant house plot inside a municipal area is Rs. 50,000. Find the yearly tax if the municipality demands tax at the rate of Rs. 6·50 for every Rs. 1000 for 6 months.

TEST PAPER 7 (2)

[Time : 45 minutes

1. Coffee seeds are exported from India to England to the value of Rs. 3,60,000. Find its value in English currency (1 Pound = Rs. 18).

2. A person insures his life at the age of 25 for Rs. 10,000 for a period of 30 years. The rate of yearly premium for every Rs. 1000 is Rs. 34·75. Find the half yearly premium to be paid.

3. A man bought 20 mathematics books at Rs. 4-50 each and 25 science books at Rs. 5-40 each and got a discount of 10% on the purchases. How much has he paid for the books ?

4. (1) Express $\frac{3}{8}$ as percentage.

(2) Convert 40% into a fraction.

(3) Find 35% of Rs. 4·60.

5. The population of a village increases every year by 10%. The present population of the village is 2200. What will be the population next year ?

6. A merchant gets a profit of 5% by selling a saree for Rs. 63. At what price should he sell the saree in order to get a profit of 20% ?

7. A person purchased a house for Rs. 50,000 and spent Rs. 10,000 on repairs. Then he lent it for a monthly rent of Rs. 600 to a tenant. He spends Rs. 1,200 every year on repairs. Find the percentage of interest that is realised on his investment.

ANSWERS

Exercise 1—1

1. (a) 10 : 3 (b) 15 : 1 (c) 4 : 1 (d) 5 : 1
(e) 25 : 2 (f) 6 : 1 (g) 9 : 25 (h) 5 : 3.

2. (a) 4 : 14 (b) 2 : 6 (c) 10 : 6 (d) 14 : 18
(e) 1 : 4 (f) 2 : 10 : 12.

3. (i) a & f (ii) b & g (iii) c & e (iv) d & h

4. (1) $\frac{2}{7}$ (2) $\frac{11}{15}$ (3) $\frac{7}{16}$ (4) $\frac{8}{12}$

5. (1) 1 : 2 (2) 3 : 4 (3) 1 : 7 (4) 4 : 9

6. 90° 7. A Rs. 245, B Rs. 105 8. (i) 1 : 3 (ii) 17 : 39
9. Rs. 135

Exercise 1—2

1. (1) 1500 m (2) 5 cm (3) 150 km (4) 3·5 cm
(5) 5 cm

3. (a) 1 : 500 (b) 1 : 50000 (c) 1 : 100000 (d) 1 : 8000

4. 40 km 5. 3·5 cm 7. 1 cm = 2·5 km

Exercise 2—1

1. (a) direct (b) direct (c) inverse (d) direct
(e) inverse (f) No (g) No (h) inverse

2. Rs. 86.40 3. Rs. 585 4. 40 5. 24 6. 12 m

7. $13\frac{1}{2}$ hrs. 8. 9 hrs.

Exercise 2—2

7. Rs. 900 8. Rs. 8400 9. Rs. 15 10. 10 days 11. 60 days

Exercise 2—3

7. 32 km 8. 50 m/sec, 3 km/min 9. 4 p.m. 10. 6.30 p.m.
11. 190 km.

Exercise 3—1

13. (a) Rs. 864 (b) Rs. 200 (c) Rs. 320 (d) Rs. 105 (e) Rs. 198
14. 2nd person, Rs. 40 15. Rs. 300 16. Rs. 2000 17. 5%
18. (a) 8% (b) 10% 19. 10 years 20. Rs. 1440
21. $5\frac{5}{12}$% 22. 2 years

Exercise 3—2

1. Rs. 130·40 2. Rs. 122·50; Rs. 215·00; Rs. 337·50; Rs. 13·50; Rs. 351 3. Rs. 8·10 4. Rs. 22,800 5. 12%
6. Rs. 385 7. 4%

Exercise 3—3

1. 80,00,000 dollars 2. Rs. 1,08,000 3. Rs. 1452
4. 18,90,000 liras 5. 40,000 dollars.

Exercise 3—4

1. Rs. 64, Rs. 576 2. 20% 3. Rs. 17,900

Exercise 3—5

1. Rs. 60 2. 3 Ps 3. Rs. 325 4. (a) Rs. 106-38
(b) (i) Rs. 422 - 48 (*ii*) Rs. 280 - 96 (c) Rs. 86-88
(d) Rs. 70 - 93 5. Rs. 630 6. $1\frac{1}{4}$%

Exercise 3-6

1. Rs. 275 2. 10 paise 3. Rs. 300 4. Rs. 2000

Exercise 3—7

9. 750 10. 3600, 1900 11. 357 12. 51, 840
13. Rs. 40,500 14. 3,16,250; 1,00,000 15. 8736
16. (1) Rs. 2.40; 40 Ps (+) (2) Rs. 2; Re. 1(+)
(3) $9\frac{1}{11}$% (−); 50 Ps (−) (4) Rs. 6; Rs. 1-50 (+)
(5) 5% (−); Rs. 20 90 17. Rs. 540; 8%

Exercise 3—8

8. Rs. 3,000 9. Rs. 4,000 10. Rs. 1,500
11. (a) Rs. 486 (b) Rs. 10,800 (c) 423.20 (d) Rs. 62.92
12. (a) Rs. 1,000 (b) Rs. 75 (c) Rs. 6.25 (d) Rs. 1,000
13. 44.8% 14. 30% 15. 2.5% loss 16. Rs. 220 17. 50%

TEST PAPER 7 (1)

1. 1 cm = 400 m 2. 15 cm 3. 6 hours 4. 7
5. 30 km/hour 6. Rs. 27,400 7. Rs. 650

TEST PAPER 7 (2)

1. 20,000 Pounds 2. Rs. 173·75 3. Rs. 202·50
4. (1) $37\frac{1}{2}$% (2) $\frac{2}{5}$ (3) Rs. 1·61
5. 2420 6. Rs. 72 7. 10

8. GEOMETRY

1 — 1. Line segment, Angles

PRE TEST

Exercise I

1. Draw four different line segments. Measure them and name them.

2. Draw a line segment $PQ = 7{\cdot}2$ cm. Draw another line segment RS congruent to PQ.

3. How many lines can be drawn through 3 points not in a line by taking two at a time? Draw a diagram to illustrate the answer.

4. Draw a right angle approximately without using a protractor, and measure it.

5. Construct the following angles. What kind of angles are they ?

   48, 60, 120, 75, 123, 135, 148, 180.

6. Measure the following angles :

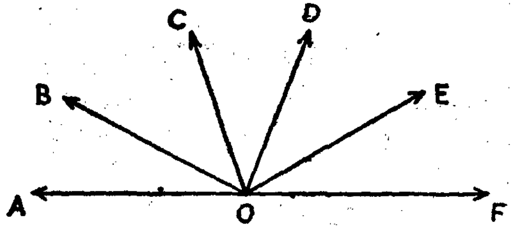


Fig. 8-1.

$m\angle FOD$ =	$m\angle AOD$ =
$m\angle FOE$ =	$m\angle AOE$ =
$m\angle BOE$ =	$m\angle AOC$ =
$m\angle BOD$ =	$m\angle COD$ =
$m\angle AOB$ =	$m\angle COF$ =

1—2. Line, Point, Angle (Revision)

Let us recollect what we have studied in the previous classes. A point may be taken to represent a geometrical point. Even if we mark a dot with a sharp point of a pencil on a sheet of paper it is just a representation and actually not a point.

A line stretches endlessly on both sides.

Fig. 8-2.

Fig. 8-3.

A ray has a starting point but it has no end. The starting point of the ray is also called the end point of the ray.

A line segment has a beginning as well as an end point. Both points are called the end points of the segment.

Fig. 8-4.

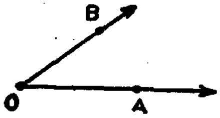


Fig. 8-5.

Two non - collinear rays with a common end point determine an angle.

When the rays of an angle form a rectangular corner, it becomes a right angle, whose measure is 90°.

If the measure of an angle is less than 90° it is called an acute angle.

The measure of the angle formed by two opposite rays is 180°.

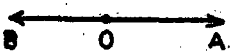


Fig. 8-6.

2. Angles

2—1. Complementary Angles

Two angles are called **complementary angles** if the sum of their measures is 90°. The complement of 50° is 40°. The complement of 30° is 60°.

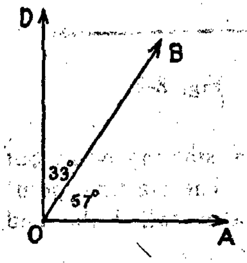


Fig. 8-7.

In Fig. 8-7, the sum of the measures of $\angle AOB$ and $\angle BOD$ is a right angle.

$m\ \angle AOB + m\ \angle BOD = 1$ right angle. Here one is the complement of the other. They are complement to each other.

Complement of $\angle AOB$ is $\angle BOD$; Complement of $\angle BOD$ is $\angle AOB$.

2—2. Supplementary Angles

Two angles are said to be **Supplementary angles** if the sum of their measures is 180°. The supplement of 40° is 140°. The supplement of 110° is 70°.

40° and 140° are supplementary angles.

110° and 70° are supplementary angles.

The supplement of 80° is 100°.

2—3. Adjacent Angles

In Fig. 8-8, $\angle ABC$ and $\angle CBD$ have a common ray BC. They lie on opposite sides of the common ray BC and are formed by $\overrightarrow{AB}$ and $\overrightarrow{BD}$. These angles are called adjacent angles.

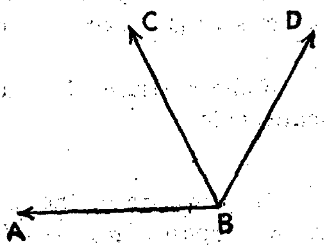


Fig. 8-8.

2—4. Linear Pair

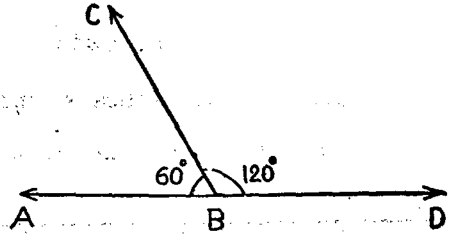


Fig. 8-9.

ABD is a line. *BA* and *BD* are opposite rays. *BC* is another ray starting from *B*. These rays form $\angle ABC$ and $\angle DBC$.

$$m\angle ABC + m\angle DBC = 180^\circ$$

Such adjacent angles whose sum of the measures is equal to 180° are said to form a **Linear Pair**. Linear comes from the word line. Since two of the rays of these angles, namely $\overrightarrow{BA}$ and $\overrightarrow{BD}$ are the opposite rays they get the name linear pair.

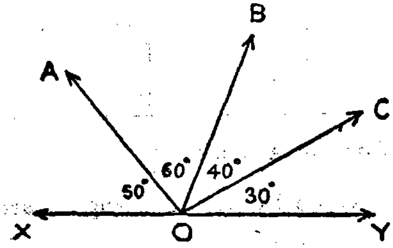


Fig. 8-10.

In Fig. 8-10, many rays have been drawn from a point on the same side of a line. Discover that the sum of the measures of angles which are formed by these rays is equal to 180°.

2—5. Vertically Opposite Angles

In Fig. 8 - 11, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at *O*. Now we have the rays *OA*, *OB*, *OC*, *OD* all having the same starting point, namely *O*. Four angles, $\angle AOC$, $\angle COB$, $\angle BOD$ and

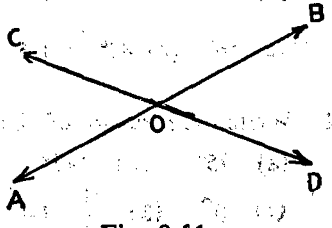


Fig. 8-11.

$\angle DOA$ are formed. Now measure the four angles using a protractor.

(i) $\angle AOD$, $\angle BOC$ (ii) $\angle BOD$, $\angle AOC$.

These pairs of angles are called vertically opposite angles.

Draw similar figures, measure their angles and tabulate them below:

Fig. No.	$m\angle AOD$	$m\angle BOC$	$m\angle BOD$	$m\angle AOC$
1.				
2.				
3.				
4.				

What do we understand from the above table?

We have discovered that vertically opposite angles are congruent.

$$m\ \angle AOC = m\ \angle BOD$$
$$m\ \angle BOC = m\ \angle AOD$$

Exercise 2

1. Find the measure of the complement of the following:

(a) 16° (b) 33° (c) 52° (d) $62\frac{1}{2}°$ (e) 80° (f) $x°$ (g) 3°

2. Write the measure of the supplement of the following :

(a) 75° (b) 143° (c) 32° (d) 54° (e) 112°
(f) p° (g) $\frac{1}{2}$ right angle.

3. Fill in the blanks suitably :

(a) The supplement of a right angle is ________

(b) The supplement of an obtuse angle is ________

(c) The supplement of an acute angle is ________

4. If two complement angles are congruent what is the measure of each ?

5. If the measure of one angle is twice that of its complement, what is the measure of each ?

6. The measure of one of the supplementary angles is half the measure of the other. Find the measure of each.

7. In Fig 8-12, if
$m\angle PQS = m\angle TQR$,
$m\angle PQT = 80°$,
$m\angle SQT = 50°$,
find $m\angle PQR$.

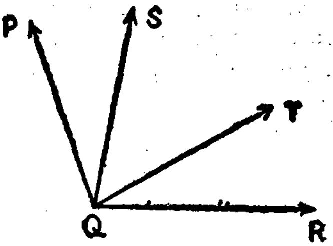


Fig. 8-12.

8. Write down the pairs of vertically opposite angles in Fig. 8-13.

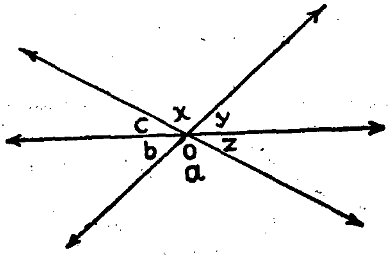


Fig. 8-13.

9. Look at Figs. 8-14 (i) and (ii). How many pairs of vertically opposite angles are formed in each? What are they ?

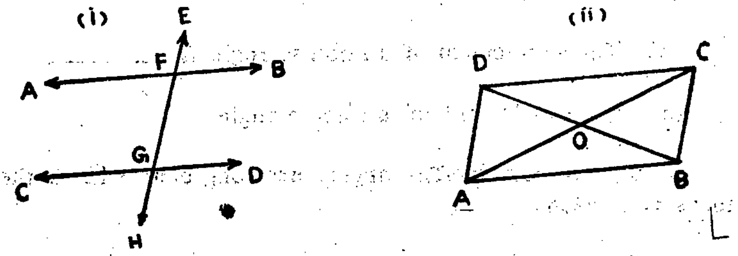


Fig. 8-14.

10. In Fig. 8-15, lines *AB*, *CD*, *EF* and *GH* intersect at *O*. If *X* is a right angle, prove that a + d + e = *X*.

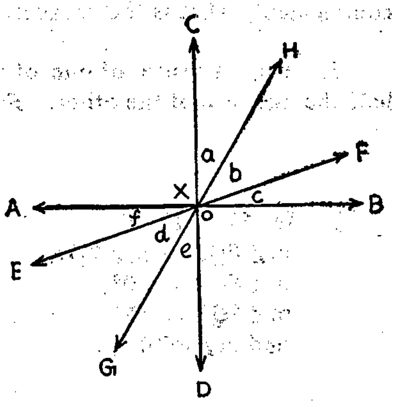


Fig. 8-15.

11. Find the measures of the remaining angles.

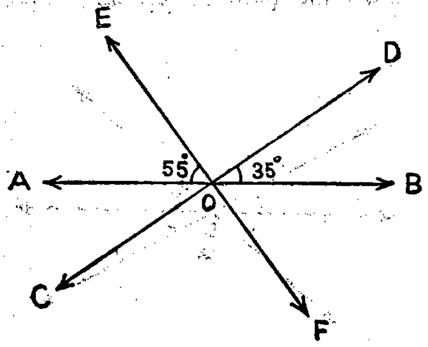


Fig. 8-16.

12. m $\angle COE$ = ?

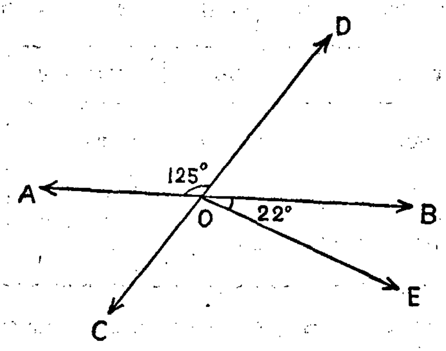


Fig. 8-17.

3. Triangles

3—1. Kinds of triangles

PRE—TEST

Exercise 3 (a)

1. How are triangles classified according to the lengths of their sides ?

2. How are triangles classified according to the measures of their angles ?

3. Tell what kind of triangle each of the following is :

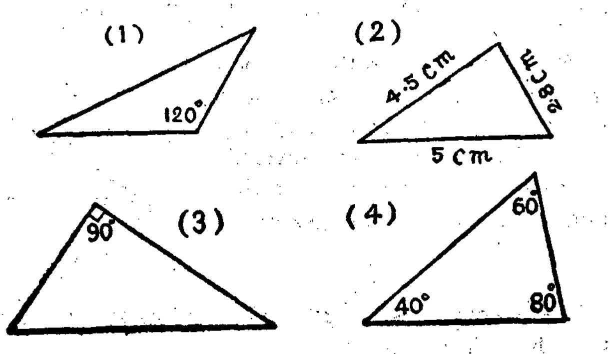


Fig. 8-18.

4. The perimeter and the lengths of two sides of the triangles are given below. Find the length of the third side.

S.No.	Perimeter	1st side	2nd side	3rd side
1.	15 cm	5 cm	4 cm	....
2.	24 cm	7 cm	9 cm	....
3.	20 cm	5 cm	7·5 cm	....

5. Write the names of the vertices and the sides opposite to them in each of the following triangles :

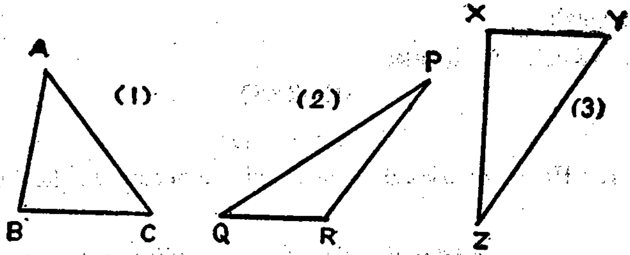


Fig. 8-19.

3—2.

Let us recall what we have learnt previously.

Plot three non-collinear points *A*, *B*, *C* on a sheet of paper. Join the points by three line segments *AB*, *BC* and *CA*. The closed figure formed by the three line segments is called a triangle. It is denoted by $\triangle ABC$.

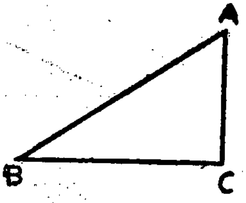


Fig. 8-20.

The triangle has *AB*, *BC* and *CA* as its three sides. Note that it has three angles. They are $\angle A$ or $\angle BAC$, $\angle B$ or $\angle ABC$, $\angle C$ or $\angle ACB$.

In a triangle each pair of sides has a common end point. In Fig. 8-20, the common end point of the line segments *AB* and *AC* is *A*. This is called a vertex of the triangle. Side *BC* is opposite to this vertex. A triangle has 6 parts such as three sides and three angles.

3—3. Triangle — its base and height

In triangle *ABC*, $\overline{BC}$ is its base. From the vertex *A* the altitude *AD* is drawn to $\overline{BC}$ and it is known as the height of the triangle. Let us consider any side of a triangle as its base. The perpendicular line drawn to it from its opposite vertex is called its height.

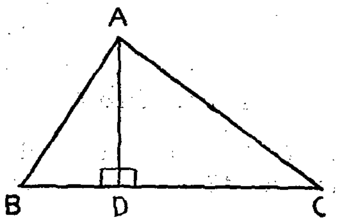


Fig. 8-21.

In triangle *ABC*, $\angle ACB$ is an obtuse angle. $\overline{BC}$ is its base. To know its height draw a perpendicular to the extension of *BC* from the vertex *A*. Measure $\overline{AD}$

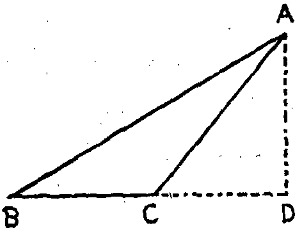


Fig. 8-22.

In a particular triangle, one of its sides may become its altitude. What kind of triangle is it? Draw a model figure.

3—4. Equilateral Triangle

Measure the lengths of the sides and angles of triangles *PQR* and *LMN* (Fig. 8-23).

If all the three sides are congruent, the triangle is called an equilateral triangle. We will also find that all the three angles of an equilateral triangle are congruent. The measure of each angle is 60°.

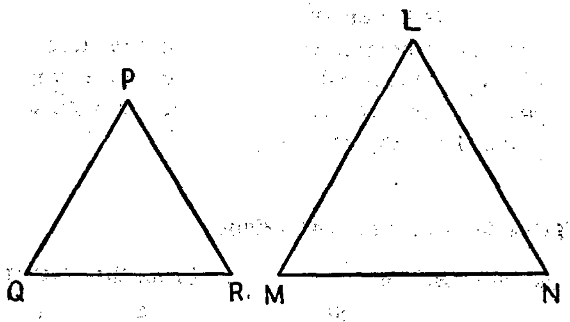


Fig. 8-23.

3—5. Isosceles Triangle

Find the measures of the three sides and three angles in triangles *ABC* and *PQR* (Fig. 8-24). What do you note?

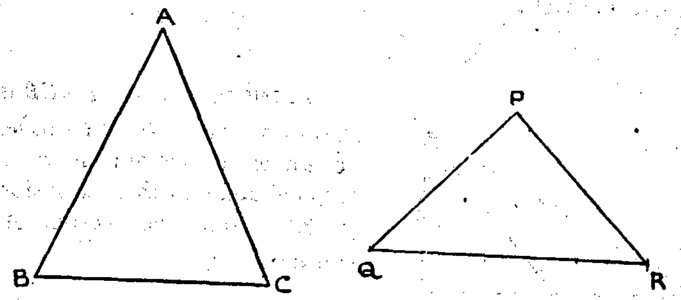


Fig. 8-24.

Triangles having two sides congruent are called isosceles triangles. The measures of angles opposite to the congruent sides are also congruent. Likewise if any two angles of a triangle are congruent, then the sides opposite to them are also congruent.

3—6. The sum of the measures of the three angles of a triangle

The sum of the measures of the three angles of a triangle is 180°.

Practical Work (a): (Paper cutting)

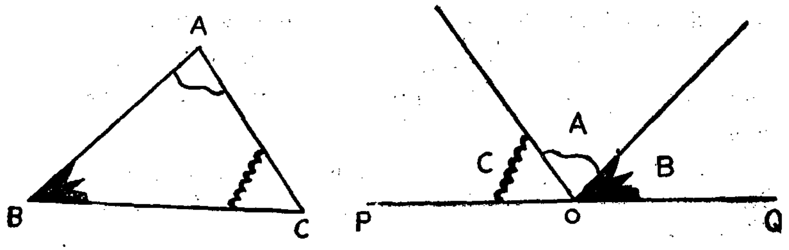


Fig. 8-25.

Cut out the three angles of the triangle *ABC*. Paste them on *PQ* as shown in the figure. We can find that all the three angles form a line. We have already known that the sum of the measures of adjacent angles lying on a line is 180°.

$$\therefore \; m\angle A + m\angle B + m\angle C = 180°.$$

Practical Work (b): (By drawing triangles)

Draw five triangles and find the measures of the angles. Tabulate them as follows:

Fig. No.	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle A + m\angle B + m\angle C$
1				
2				
3				
4				
5				

What do you find about the sum of the measures of the three angles?

Remember

The sum of the measures of the three angles of a triangle is 180°.

Exercise 3(b)

1. If the measure of the vertex angle of an isosceles triangle is 80° find the measures of the other angles.

2. The measure of one of the congruent angles of an isosceles triangle is 70°. Find the measures of the other angles.

3. The perimeter of an isosceles triangle is 12 cm. The length of one of the equal sides is 3·5 cm. What is the length of each of the other two sides ?

4. The perimeter of an isosceles triangle is 14 cm and its base is 4 cm. Find the length of each of the congruent sides.

5. In an isosceles triangle the measure of one of its congruent angles is given below. Find the measures of the other angles.

(1) 80° (2) 64° (3) 50° (4) 35°

6. In an isosceles triangle, the measure of the vertex angle is given. Find the measure of the congruent angles.

(1) 120° (2) 70° (3) 34° (4) 90°

4. Quadrilaterals

4—1. Kinds of four sided figures :

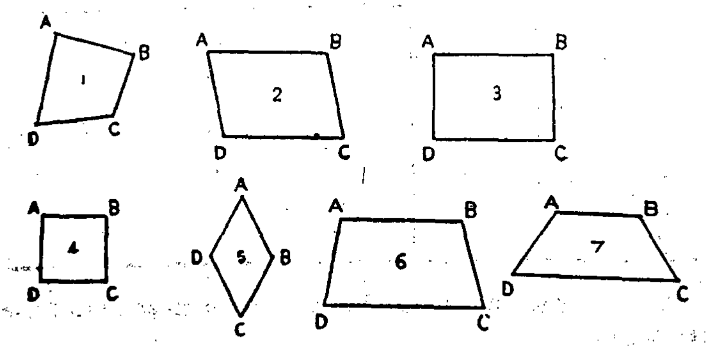


Fig. 8-26.

If four non-collinear points *A*, *B*, *C*, *D* in a plane form a closed figure by the line segments, *AB*, *BC*, *CD* and *DA*, then the closed figure is called a quadrilateral.

4—2. The diagonals of a quadrilateral

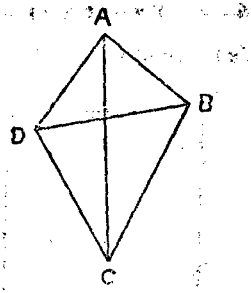


Fig. 8-27.

ABCD is a quadrilateral. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ are its sides. $\overline{AC}$ and $\overline{BD}$ are its diagonals. A quadrilateral has two diagonals which intersect at a point.

4—3. Angles of a quadrilateral

In quadrilateral *ABCD* the diagonal *BD* is drawn. The quadrilateral is divided into two triangles, namely *ABD* and *BCD*. We know the sum of the measures of the three angles of a triangle is 180°.

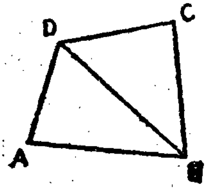


Fig. 8-28.

∴ The sum of the measures of the four angles of a quadrilateral *ABCD* = the sum of the measures of the six angles of the two triangles

$$=180°+180°=360°.$$

Verify :

Draw some quadrilaterals, measure the angles and tabulate them as follows :

Quadri-lateral No.	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle D$	$m\angle A+m\angle B+m\angle C+m\angle D$
1.					
2.					
3.					
4.					

What do you find from the last column of the table ?

The sum of the measures of the angles of a quadrilateral s 360°.

4—4. Properties of special quadrilaterals

(a) Square :

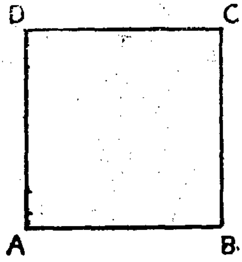


Fig. 8—29.

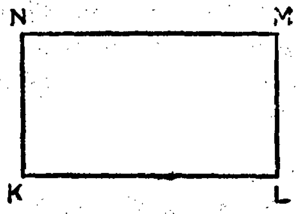


Fig. 8—30.

We have already studied about the above figures in the previous classes. Measure their sides and angles. Draw their diagonals and measure them.

List out the properties of line segments, angles and diagonals.

S. No.	Name of the shape	Properties of sides	Properties of angles	Properties of diagonals
1.				
2.				

We find :

In a square :

1. The sides are congruent.

2. The angles are congruent and the measure of each is 90°.

3. The lengths of the diagonals are equal.

In a rectangle :

1. The lengths of opposite sides are equal and the lengths of adjacent sides are not equal.
2. The measures of the angles are equal and each is 90°.
3. The diagonals are of equal length.

(b) Parallelogram :

ABCD is a parallelogram. Here the opposite angles and opposite sides are congruent. Opposite sides are parallel to each other. Therefore we call it a parallelogram.

$\overline{AB} = \overline{CD}$

$\overline{BC} = \overline{DA}$

$m\angle A = m\angle C$

and $m\angle B = m\angle D$

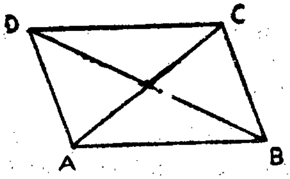


Fig. 8-31.

Here the adjacent sides namely $\overline{AB}$ and $\overline{AD}$ are not equal in length. $\overline{AC}$ and $\overline{BD}$ are diagonals. The diagonals of a parallelogram are not equal in length. In a parallelogram, the sum of the measures of any two adjacent angles is 180°.

$m\angle A + m\angle B = 180^\circ$; $m\angle A + m\angle D = 180^\circ$.

(c) Rhombus :

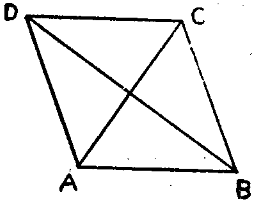


Fig. 8-32.

A rhombus is a parallelogram having all the sides equal in length.

$AB = BC = CD = DA$.

AC and *BD* are diagonals. They are not equal in length. But they intersect at right angles.

Practical Work :

Fig. 8-33.

Take four frames of equal length and join them by means of nails. Now tilt the position of the corners so that the measures of the angles at each corner is not equal to 90°. The figure thus formed is a **Rhombus.**

Similarly a parallelogram may be obtained by tilting the position of a rectangular frame.

(d) Trapezium :

In a quadrilateral if only one pair of opposite sides are parallel, then it is called a Trapezium

In Trapezium *ABCD*, *AB* and *CD* are parallel; *BC* and *AD* are not parallel.

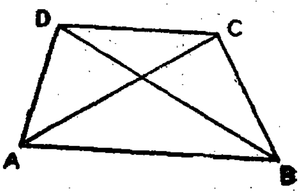


Fig. 8-34.

AC and *BD* are diagonals.

(e) Family of Quadrilaterals :

In Fig. 8-35 note the relation between the figures.

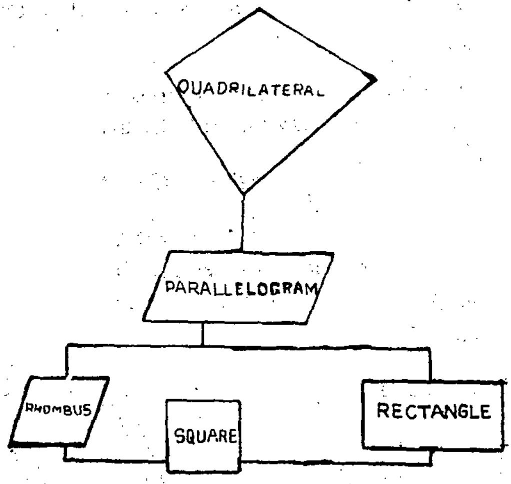


Fig. 8-35.

Remember

S. No.	Figure	Properties of sides	Properties of angles	Properties of diagonals
1.	Square	All sides are congruent; opposite sides are parallel.	All angles are congruent. Each angle is a right angle.	Equal in length. Intersect at right angles.
2.	Rectangle	Only opposite sides are congruent and parallel.	All angles are congruent. Each angle is a right angle.	Equal in length. Bisect each other.
3.	Parallelogram	Only opposite sides are congruent and parallel.	Pairs of opposite angles are congruent.	Not equal in length. Bisect each other.
4.	Rhombus	All the four sides are congruent. Pairs of opposite sides are parallel.	Pairs of opposite angles are congrueut.	Not equal in length. Bisect at right angles.
5.	Trapezium	One pair of opposite sides are parallel.		

Exercise 4

1. Put '√' for true statements and '×' for false statements in the following table :

S. No.	Properties	Parallelogram	Rhombus	Rectangle	Square	Trapezium
1.	Measures of all angles are equal					
2.	Diagonals are equal in length					
3.	Only opposite angles are equal in measure					
4.	All sides are congruent					
5.	Only opposite sides are congruent					
6.	A pair of opposite sides are congruent					
7.	Adjacent sides are congruent					

2. List out the similarities and the differences between a rectangle and a parallelogram.

3. List out the similarities and the differences between a square and a rhombus.

4. List out the similarities and the differences between a square and a rectangle.

5. Practical Geometry

Construction of triangles

To draw a triangle, three independent measures are needed. Atleast the measure of one side should be given. The triangle cannot be drawn if the measures of three angles alone are given.

5—1. Construction of a triangle when three sides are given

A triangle can be drawn only if the sum of the lengths of any two sides is greater than the third side.

Example :

Construct a triangle *ABC* where $\overline{AB} = 6$ cm; $\overline{BC} = 4$ cm; $\overline{CA} = 5$ cm.

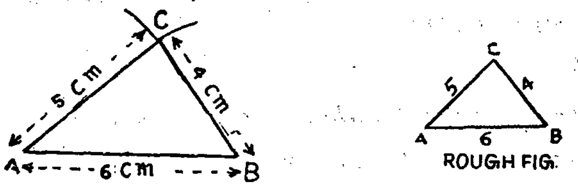


Fig. 8-36.

Procedure :

1. First draw $\overline{AB} = 6$ cm. Now you have got the vertices *A* and *B* of $\triangle$ *ABC*. Let us now find the vertex *C*.

2. With *A* as centre and radius 5 cm draw an arc above *AB*.

3. With *B* as centre and radius 4 cm, draw another arc to cut the previous arc at *C*.

4. Draw AC and BC.

5. ABC is the required triangle. (Note that another triangle on the other side of AB also can be drawn with the same measures.)

Hint:

When we begin a geometrical construction, we should draw a rough figure and note the given measurements in it.

Exercise 5—1

Construct triangles with the following measurements.

1. In $\triangle ABC$, $\overline{AB} = 5$ cm; $\overline{BC} = 6$ cm; $\overline{AC} = 8$ cm.
2. In $\triangle PQR$, $\overline{PQ} = 6$ cm; $\overline{QR} = 7$ cm; $\overline{PR} = 5{\cdot}8$ cm.
3. In $\triangle LMN$, $\overline{MN} = 8{\cdot}4$ cm; $\overline{LM} = 7{\cdot}5$ cm; $\overline{LN} = 6{\cdot}6$ cm.

Measure the three angles in each of the triangles.

5—2. Construction of a triangle when two sides and the included angle are given

Example :

Construct a triangle ABC, where $AB = 6$ cm ;

$\overline{AC} = 4$ cm ; $m\angle A = 50°$.

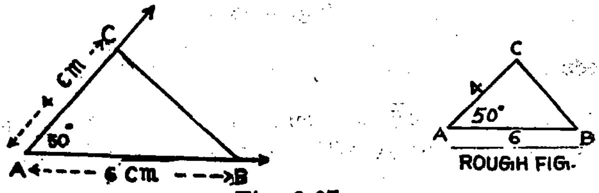


Fig. 8-37.

Procedure :

1. Mark a point A.
2. Construct an angle whose measure is $50°$ at A.

3. Let one of the arms of the angle be AX and the other arm be AY.

4. Mark B and C on it such that $\overline{AB} = 6$ cm and $\overline{AC} = 4$ cm.

5. Draw BC

6. ABC is the required triangle

(Another triangle can be drawn on the other side of AB with the same measures.)

Exercise 5—2

Construct triangles with the following measurements:

1. In $\triangle ABC$, $\overline{BC} = 7{\cdot}5$ cm ; $\overline{AB} = 7$ cm ; $m\angle B = 80°$.

2. In $\triangle DEF$, $\overline{DE} = 6{\cdot}5$ cm ; $\overline{EF} = 5{\cdot}6$ cm ; $m\angle E = 60°$.

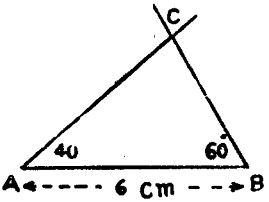
3. In $\triangle XYZ$, $\overline{YZ} = 8$ cm ; $\overline{XZ} = 5{\cdot}8$ cm ; $m\angle Z = 100°$.

Measure the other two angles of the triangles constructed.

5—3. To construct a triangle given two angles and one side

Example :

Consruct a triangle ABC where $\overline{AB} = 6$ cm; $m\angle A = 40°$; $m\angle B = 60°$.



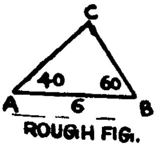


Fig. 8–38.

1. Draw $\overline{AB} = 6$ cm.

2. Construct an angle of measure 40° at A and of measure 60° at B.

3. The intersecting point of the arms of these angles is C.

4. ABC is the required triangle.

If $m\angle C$ is given instead of $m\angle B$ or $m\angle A$, first find $m\angle B$ or $m\angle A$ by calculation and then construct the triangle.

Another triangle also can be constructed with the same measures on the other side of AB.

Exercise 5—3

Construct triangles with the following measurements:

1. In $\triangle KLM$, $\overline{LM} = 8$ cm; $m\angle L = 60^\circ$; $m\angle M = 70^\circ$.

2. In $\triangle RST$, $\overline{RS} = 7{\cdot}5$ cm; $m\angle S = 75^\circ$; $m\angle T = 35^\circ$.

3. In $\triangle XYZ$, $\overline{YZ} = 5{\cdot}5$ cm; $m\angle Y = 110^\circ$; $m\angle Z = 40^\circ$.

Measure the other two sides of each triangle.

5—4. Construction of Equilateral Triangle

Each of the sides of an equilateral triangle is $4{\cdot}5$ cm. Construct the triangle.

All its three sides are equal in length. We have already studied how to construct triangles given three sides. The same method may be followed to construct an equilateral triangle.

Exercise 5—4

1. The length of the side of an equilateral triangle is $5{\cdot}5$ cm. Construct the triangle and measure the angles.

2. Construct an equilateral triangle ABC in which $\overline{BC} = 5{\cdot}6$ cm. Draw an altitude and find its height.

3. Construct an equilateral triangle PQR in which $QR = 7{\cdot}5$ cm.

5—5. Construction of right angled triangle

We know already that the measure of one of the angles of a right angled triangle is 90°. The side opposite to it is called the **hypotenuse.**

Let us see the construction of a right angled triangle when the sides containing the right angle are given.

Construct $\triangle ABC$ in which $\overline{BC} = 5$ cm, $\overline{AB} = 7$ cm and $m\angle B = 90°$.

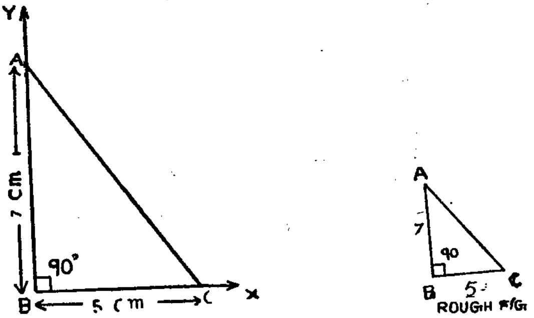


Fig. 8-39.

Procedure :

1. Mark point B and draw an angle of measure 90° at B.

2. Draw the arms BX and BY and cut an arc on $\overrightarrow{BX}$ with $\overline{BC} = 5$ cm and another arc on $\overrightarrow{BY}$ with $\overline{BA} = 7$ cm.

3. Draw AC.

4. ABC is the required triangle.

Exercise 5—5

Construct right triangles with the following measurements. Measure the other angles and sides.

1. $\overline{AB} = 7{\cdot}5$ cm ; $\overline{BC} = 4{\cdot}8$ cm ; $m\angle B = 90°$.

2. $m\angle C = 90°$; $\overline{AC} = 8{\cdot}9$ cm ; $\overline{BC} = 8$ cm.

3. $m\angle C = 90°$; $\overline{BC} = 5{\cdot}8$ cm ; $\overline{AC} = 4{\cdot}2$ cm.

4. $\overline{BC} = 6$ cm ; $\overline{AC} = 10$ cm ; $m\angle C = 90°$.

5. $\overline{BC} = 5{\cdot}4$ cm ; $m\angle B = 90°$; $m\angle C = 55°$.

5—6. To bisect a given line segment

(a) Paper folding

Fold a sheet of paper along the horizontal dotted lines as shown in the figure. You can get a line segment along the

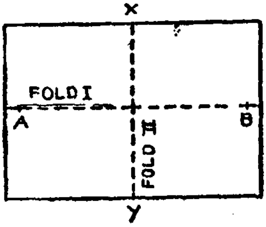


Fig. 8—40.

paper folding. Mark A and B on it. AB is a line segment. To bisect AB, fold the paper vertically so that B coincides with A. Now we get another line segment along the paper folding. Name it as XY. XY bisects AB.

(b) To bisect the given line segment using a pair of compasses

Example:

Draw a line segment $AB = 5$ cm and bisect it.

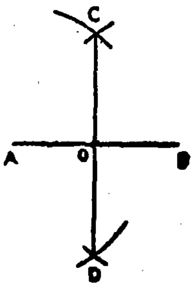


Fig. 8-41.

Procedure :

1. Draw a line segment $AB = 5$ cm.

2. With A as centre and with radius more than half of the length of AB, draw arcs on both sides of AB.

3. With B as centre draw arcs on both sides of AB with the same radius so as to cut the arcs already drawn.

4. The arcs intersect at C and D.

5. Join CD.

6. Let $\overline{CD}$ intersect $\overline{AB}$ at O.

7. $\overline{AO} \equiv \overline{OB}$; $\overline{OC} \equiv \overline{OD}$. Check these measures with dividers.

Exercise 5—6

1. Draw the line segments of the following measures and bisect them :

(a) 4·9 cm (b) 6·7 cm (c) 5·6 cm (d) 9·7 cm

2. Draw a line segment of length 11 cm and divide it into four equal parts using a pair of compasses.

5—7. To bisect an angle into two equal parts

(a) Paper folding :

Mark a point O on a sheet of paper. Draw $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Now we get $\angle AOB$. Fold the paper in such a way that $\overrightarrow{OA}$ falls on $\overrightarrow{OB}$. Name the folding as OC. Now $\overrightarrow{OC}$ bisects $\angle AOB$.

$m\angle AOC = m\angle BOC$.

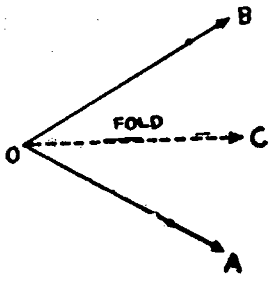


Fig. 8-42.

(b) Using a pair of compasses

Let XOY be the given angle. With O as centre draw an arc cutting the arms OX and OY at A and B. Keeping these points as centres, draw two arcs opposite to O with the same radius, to intersect each other. The point of intersection is C.

Join OC. $\overrightarrow{OC}$ is the bisector of $\angle XOY$.

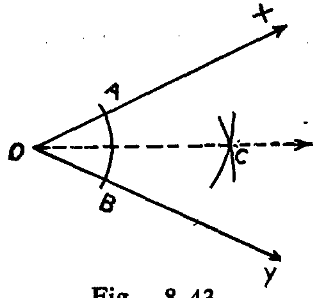


Fig. 8-43.

Measure and verify whether $m\angle XOC = m\angle YOC$.

Exercise 5—7

1. Construct the angles with the given measures. Bisect them with the help of compasses.

(a) 39° (b) 105° (c) 78° (d) 120°.

2. Costruct the angles of the following measures and divide them into four equal parts. Measure the angles and verify them.

(a) 80° (b) 140°.

5—8. Construction of a parallelogram

Three measurements are needed to construct a parallelogram.

Example:

Construct a parallelogram $ABCD$ in which $\overline{AB} = 5{\cdot}5$ cm, $\overline{AD} = 4$ cm and $m\angle A = 50°$.

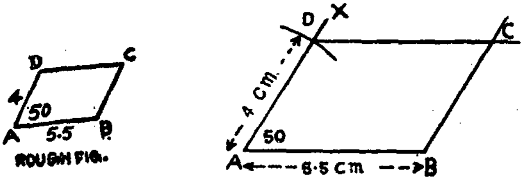


Fig. 8-44.

Procedure :

1. Draw $\overline{AB} = 5{\cdot}5$ cm.
2. Construct an angle, $\angle BAX$ of measure 50°.
3. Mark D on $\overrightarrow{AX}$ such that $\overline{AD} = 4$ cm.
4. Draw line segments parallel to $\overline{AB}$ and $\overline{AD}$.
5. Their intersection is C.
6. $ABCD$ is the required parallelogram. Measure $\overline{BC}$ and $\overline{DC}$ and measure the other angles also.

Exercise 5—8

Construct the parallelogram with the following measurements. Measure the other angles and sides.

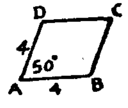
1. $\overline{AB} = 7$ cm; $\overline{BC} = 4{\cdot}5$ cm; $m\angle B = 52^\circ$
2. $\overline{CD} = 6$ cm; $\overline{BC} = 3{\cdot}2$ cm; $m\angle C = 125^\circ$
3. $\overline{CD} = 8$ cm; $\overline{AD} = 4{\cdot}8$ cm; $m\angle D = 70^\circ$
4. $\overline{AB} = 6{\cdot}6$ cm; $\overline{AD} = 5{\cdot}4$ cm; $m\angle A = 110^\circ$

5—9. Construction of Rhombus

We have already known that a rhombus is a parallelogram with adjacent sides congruent. To construct a rhombus we need two measurements.

Example :

Construct a rhombus whose side is 4 cm and the measure of an angle is 50°.



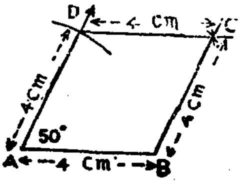


Fig. 8-45.

We have just now learnt how to draw a parallelogram. In the same way a rhombus can be drawn. Measure the angles and sides. Verify the properties of a rhombus.

Exercise 5–9

Construct the rhombus *ABCD* with the following measurements :

1. $\overline{BC} = 6$ cm ; $m\angle B = 45^\circ$

2. $\overline{AB} = 7{\cdot}5$ cm; $m\angle B = 120^\circ$

3. $m\angle A = 70^\circ$; $\overline{AB} = 5{\cdot}5$ cm

4 $m\angle A = 100^\circ$; $\overline{AB} = 6{\cdot}4$ cm.

5—10. Concentric Circles

We have already learnt about the circle and its parts. We shall now learn a few more facts about circles.

Circles with the same point as centre are called **concentric circles.**

Look at the figure. Note that the circumferences of these circles do not cut or touch each other; they are at the same distance apart at all places. The region between the two circles is called a circular ring. The cross section of a pipe, circular paths, cross section of a ring, washer etc. are examples of circular rings.

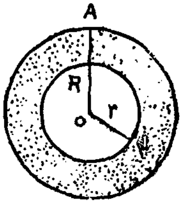


Fig. 8-46.

In Fig. 8-46, *OA* is the radius of the outer circle or bigger circle and is equal to *R* units. *OB* is the radius of the smaller circle. *It is equal to r units.*

Hence the width of the circular ring = (*R* – *r*) units.

Exercise 5—10

1. Construct the concentric circles with inner and outer radii equal to 2·5 cm and 4 cm respectively. Measure the width of the circular ring.

2. Construct three concentric circles with the radii equal to 4 cm, 6 cm and 8 cm respectively.

5—11. Touching Circles and Intersecting Circles

1. Two circles each of whlch is outside the other.

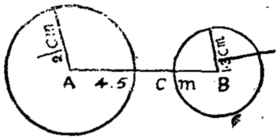


Fig. 8 - 47.

2. Two circles one of which is inside the other but do not cut each other.

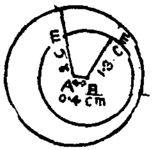


Fig. 8 - 48.

3. Externally touching circles.

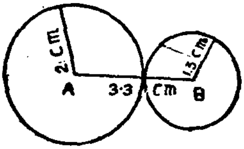


Fig. 8 - 49.

4. Internally touching circles.

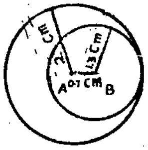


Fig. 8—50.

5. Intersecting circles.

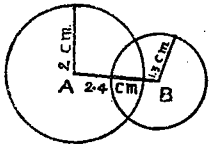


Fig. 8 - 51.

In the above figures, find the distance between the centres of the pairs of circles and the sum or difference of the lengths of their radii. Examine and discover the following relationships among them :

1. The distance between the centres of two externally touching circles is equal to the sum of the lengths of their radii. (Fig. 8-49)

2. The distance between the centres of two circles which touch internally is equal to the difference of the lengths of their radii. (Fig. 8-50)

3. The distance between the centres of two non-intersecting circles is greater than the sum of their radii. (Fig. 8-47)

4. The distance between the centres of two circles, one of which is inside the other but not cutting each other, is less than the difference of their radii. (Fig. 8-48)

5. **The distance between the centres of concentric circles is zero. (Fig. 8-46)**

6. The distance between the centres of two intersecting circles is less than the sum of their radii. (Fig. 8-51)

Exercise 5—11

1. Draw circles satisfying the following conditions. Measure the lengths of their radii and the distance between their centres.

   (a) A pair of circles touching each other externally

   (b) A pair of circles touching each other internally

   (c) Circles intersecting each other.

2. Draw two circles whose radii are of lengths 3 cm and 2 cm respectively and whose centres are 4 cm apart from each other. What kind of circles are they ?

3. Draw $\overline{AB} = 5$ cm. Draw two circles one with A as centre and radius of length 3 cm and the other with B as centre and radius of length 2 cm. What do you notice ? State the reason.

4. Draw $\overline{AB} = 3$ cm. Draw two circles with its end points as their centres and having radii of lengths 6 cm and 2 cm respectively. What do you find ?

5. Draw $\overline{PQ} = 6{\cdot}5$ cm. Draw two circles with P and Q respectively as centres and with radii equal to 3 cm and 2·5 cm respectively. How are these circles situated in relation to each other ?

6. **Draw $\overline{AB} = 3$ cm. Draw circles with A and B as centres and with radii of lengths 5 cm and 2 cm respectively. To which kind do these circles belong ? State reasons.**

5—12. (a) To divide a line segment into equal parts

Draw a line segment AB and divide it into 5 equal parts.

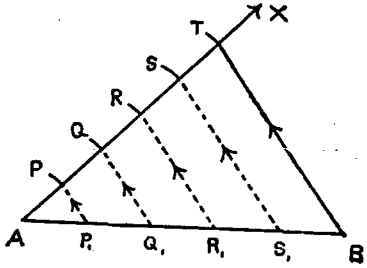


Fig. 8—52.

Procedure :

1. Draw $\overrightarrow{AX}$ passing through A and forming an acute angle with $\overline{AB}$.

2. Taking any convenient length in the compasses cut off 5 equal segments AP, PQ, QR, RS and ST on AX.

$\therefore \quad AP = PQ = QR = RS = ST.$

3. Join BT. Draw parallels $\overleftrightarrow{SS_1}$, $\overleftrightarrow{RR_1}$, $\overleftrightarrow{QQ_1}$ and $\overleftrightarrow{PP_1}$ parallel to $\overline{BT}$ through S, R, Q and P respectively.

4. These parallel lines meet AB at the points S_1, R_1, Q_1 and P_1 respectively.

5. Now AB is divided into five equal parts.

$AP_1 = P_1Q_1 = Q_1R_1 = R_1S_1 = S_1B.$

6. Verify this with the help of dividers.

5—12. (b) To divide a line segment in a given ratio

Divide the line segment AB in the ratio 4 : 3.

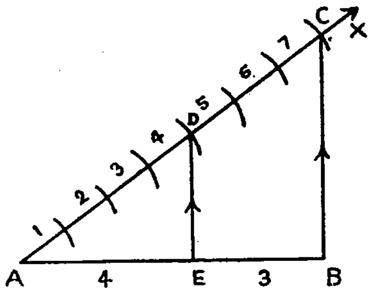


Fig. 8—53.

1. Draw $\overrightarrow{AX}$ through A making an acute angle $\angle XAB$.

2. Using the compasses mark 7 congruent segments on AX.

3. Let AC be the length of seven segments and AD the length of 4 segments.

4. Join CB and draw a parallel line through D to $\overleftrightarrow{CB}$. Let the parallel line meet AB at E.

5. E divides AB in the ratio 4 : 3.

6. Measure AE and EB and verify.

$AD : DC = 4 : 3$

$\therefore\ AE : EB = 4 : 3.$

Exercise 5—12

1. Draw two line segments each of length 7·5 cm. Bisect one of these segments by drawing arcs using the compasses and bisect the other by the parallel line method. With the help of the dividers verify whether the parts are of equal length.

2. Draw a line segment 9 cm long. Divide it into four equal parts. Measure and verify this by using dividers.

3. Draw a line segment 7 cm long and divide it into five equal parts.

4. Draw a line segment 8 cm long and divide it in the ratio 5 : 3.

5. Draw a line segment 9·6 cm long and divide it in the ratio 2 : 3.

6. Draw a line segment PQ of length 5·5 cm. Find the point A on PQ such that $PA : AQ = 3 : 2$.

7. Draw two line segments AB and CD each of length 11 cm. Divide AB into 4 equal parts by the bisection method and divide CD into 4 equal parts by the parallel lines method. Verify all the segments by using dividers.

TEST PAPER 8

[Time : 45 minutes

1. What are vertically opposite angles ? Name a pair of vertically opposite angles in Fig. 8—54.

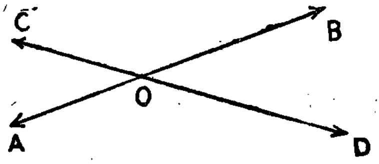


Fig. 8—54.

2. (a) What is the complement of an angle which is congruent to itself ?

(b) What is the complement of the angle of measure 82° ? Write its supplement also.

3. If the measure of the vertex angle of an isosceles triangle is 40 find the measure of each of the equal angles.

4. Write the names of the quadrilaterals whose four sides are equal.

5. What do you know about the diagonals of a rectangle and a parallelogram ?

6. The radii of two circles are 5 cm and 3 cm respectively. The distance between the centres is 6 cm. What is the relationship between them ?

7. The radii of two circles are 3 cm and 2·5 cm respectively. The centres are 5·5 cm apart. Draw the circles. What can you say about the circles ?

8. Construct a parallelogram *ABCD* with the following measurements :

$\overline{AB} = 6$ cm, $\overline{AD} = 2.5$ cm, $m\angle A = 35°$

9. Construct a triangle of sides 6 cm, 4·5 cm and 7·5 cm.

10. Construct an angle of measure 68° and bisect it using the compasses. Measure and verify the angles.

ANSWERS

Exercise 2

1. (1) 74 (2) 57 (3) 38 (4) $27\frac{1}{2}$ (5) 10 (6) $99 - x$ (7) 87

2. (a) 105 (b) 37 (c) 148 (d) 126 (e) 68 (f) $180 - p$ (g) 135

3. (a) a right angle
   (b) an acute angle
   (c) an obtuse angle

4. 45   5. 30 ; 60   6. 120 ; 60   7. 110

8. a, x ; b, y ; c, z.

9. (i) 4 pairs (ii) 2 pairs 11. $m\angle EOD = 90^\circ$
$m\angle COF = 90^\circ$; $m\angle AOC = 35^\circ$; $m\angle BOF = 55^\circ$;

12. $m\angle COE = 103^\circ$.

Exercies 3 (a)

1. 3 kinds : equilateral △ ; isosceles △ ; scalene △.

2. 3 kinds : Rt. angled △ ; obtuse angled △ ; acute angled △.

3. (1) obtuse angled △ (2) scalene △
   (3) Rt. angled △ (4) acute angled △

4. (1) 6 cm (2) 8 cm (3) 7·5 cm.

5.

	Vertex	Opp. side
(1)	A	BC
	B	CA
	C	AB
(2)	P	QR
	Q	RP
	R	PQ
(3)	X	YZ
	Y	ZX
	Z	XY

Exercise 3 (b)

1. Each 50 2. 70, 40 3. 3·5 cm, 5 cm 4. 5 cm

5. (1) 80, 20 (2) 64, 52 (3) 50, 80 (4) 35, 110

6. (1) 30 (2) 55 (3) 73 (4) 45

TEST PAPER 8

1. $A\hat{O}C$, $B\hat{O}D$; $B\hat{O}C$, $A\hat{O}D$ 2. (a) 45 (b) 8, 98

3. 70 4. Square, Rhombus

6. Intersect one another

7. Circles touch outside.

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